

# Small strain behaviour of cemented soils

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## ABSTRACT

The shear modulus of cemented soils at very small strain ( $G_0$ ) was studied. For artificially cemented clay,  $G_0$  was found to be independent of the mean effective stress until the yield stress. After yield, a significant effect of structure degradation on  $G_0$  was observed. The experimental data was interpreted by an equation, which relates  $G_0$  of cemented soils to mean stress, apparent overconsolidation ratio and the state of structure (sensitivity). The equation was also found to represent  $G_0$  of cemented sands.

## INTRODUCTION

The soil structure affects the mechanical behaviour of soil in the range from very small to large strains (Clough *et al.* 1981; Burland 1990; Leroueil and Vaughan 1990; Feda 1995; Cuccovillo and Coop 1999; Kavvas and Amorosi 2000; Cotecchia and Chandler 2000; Baudet and Stallebrass 2004). The present Note focuses on the effect of structure on the very small strain shear modulus ( $G_0$ ), when the behaviour is elastic and  $G_0$  is independent of the shear strain.

The literature review shows that natural or artificial cementation increases  $G_0$  of sands (Acar and El-Tahir 1986; Saxena *et al.* 1988; Chang and Woods 1992; Sharma and Fahey 2004) and clays (Jovičić *et al.* 2006; Puppala *et al.* 2006) in comparison with  $G_0$  of the corresponding reconstituted soil at the same mean effective stress. According to Acar and El-Tahir (1986) and Delfosse-Ribay *et al.* (2004), shear modulus of cemented sands increased with confining stress in the whole applied range. Conversely, Cuccovillo and Coop (1997), Baig *et al.* (1997), Fernandez and Santamarina (2001) and Sharma and Fahey (2004) reported  $G_0$  to be for cemented sands practically independent of the mean stress and dependent on cementation until it was reached a threshold stress corresponding to the onset of major structure degradation. Cementation appears to control only  $G_0$  of clays below isotropic or vertical yield stress and the pressure dependency appears to prevail at higher stresses accordingly (Jovičić *et al.* 2006; Hird and Chan 2008). The latter findings are consistent with the predictions of a micromechanical model for cemented granular material developed by Dvorkin *et al.* (1991), leading to the conclusion that the stiffness of the cemented system is strongly increased by cementation and independent of confining pressure.

The connection between deterioration of bonding and initiation of the pressure dependency of  $G_0$  was reported for naturally cemented carbonate sand (i.e. calcarenite; Cuccovillo and Coop 1997), loose cement treated sand (Yun and Santamarina 2005), cement treated clay (Hird and Chan 2008) and natural clay with carbonate bonding (Cafaro and Cotecchia 2001). After yielding, Cuccovillo and Coop (1997) and Cafaro and Cotecchia (2001)

reported the reduction of  $G_0$ , associated with degradation of the natural structure with increasing mean stress. Conversely, Yun and Santamarina (2005) and Hird and Chan (2008) indicated for artificially cemented soils an increase of  $G_0$  with increasing stress after yielding and the values of  $G_0$  remained higher than for the reconstituted soils.

Despite the reported differences, it can be shown that a single relationship, which relates  $G_0$  to the strength of the bonded structure, mean effective stress and apparent overconsolidation, can be used to predict the variation of  $G_0$  for three different cemented soils.

## EXPERIMENTAL DATA

### *Cemented kaolin clay*

Kaolin was mixed with distilled water at approximately the liquid limit (70%). After homogenization, Portland cement was added at the contents of 0% and 4% of the dry mass. The thoroughly mixed uniform paste was transferred into an oedometer cell or into a high consolidometer of 38 mm in diameter and consolidated at a low vertical stress of 5 kPa. After 3 days of curing (cemented soil), or after 10 days of consolidation (uncemented soil), the triaxial specimens of 76 mm height were trimmed and transferred into triaxial cells. After saturation, using the back pressure of 100 kPa, the specimens were continuously isotropically compressed up to the mean effective stress of 1500 kPa with measurements by bender elements at various stress levels. The compression rate for cemented clay was 1.25 kPa/hour. The consolidation fulfilment was controlled before each measurement. In the oedometer, the maximum vertical stress reached 7 MPa and 16 MPa during the one-dimensional compression of pure kaolin and cemented specimen, respectively.

The behaviour of the artificially cemented kaolin clay was found to be qualitatively similar to the behaviour of sensitive natural clays (Sangrey 1972; Burland 1990; Cotecchia and Chandler 1997). Figure 1(a) shows the isotropic normal compression lines (NCLs) of pure and cemented kaolin clay. Thanks to bonding, the compressibility of the cemented clay is lower until a threshold stress (of about 400 kPa), at which the cementation structure starts to degrade. The threshold isotropic state represents the maximum size of the state boundary surface (SBS). The applied stresses were not high enough to confirm the convergence of NCLs of pure and cemented clay. Oedometer tests (Figure 1(b)) however indicate that the NCLs of cemented and pure clay ultimately converge at high stresses.

Figure 2 shows the relationship between  $G_0$  and the mean effective stress  $p'$  for both types of clay specimens. The cementation increases  $G_0$  in the whole measured stress range. For pure kaolin  $G_0$  is seen to vary with  $p'$ . The values of  $G_0$  of the cemented specimen, instead, appear to be controlled by cementation, and do not vary with  $p'$ , until  $p'$  is about 400 kPa. At higher stresses,  $G_0$  increases with  $p'$ .

#### *Cemented sands*

Cuccovillo and Coop (1997) studied the behaviour of calcarenite. Results from the isotropic compression of intact and reconstituted calcarenite are shown in Figure 3(a). Undrained shearing probes were performed and the shear moduli were measured using LVDTs. According to the authors, after accounting for the compressibility of the pore fluid, the effective stress paths followed during undrained shearing were consistent with the material having isotropic properties. Figure 3(b) compares the values of shear moduli for the intact and reconstituted soil.

Yun and Santamarina (2005) tested pure and artificially cemented Nevada sand. The small strain shear modulus was measured using shear wave propagation within an oedometer cell. The data for loose uncemented sand and loose sand with 2% and 4% of cement was used in this study. The changes in specific volume are shown in Figure 4(a). The values of  $G_0$  were evaluated from the values of shear wave velocities, considering  $\rho_s = 2.67 \text{ g/cm}^3$  (Figure 4(b)).

#### QUANTIFICATION OF THE SOIL STRUCTURE

The experimental data indicates that  $G_0$  of cemented soils depends significantly on the state of soil structure. The structure can be quantified by the variable *stress sensitivity ratio*  $s$  (Cotecchia and Chandler 2000), defined as the ratio of the size of the SBS of the cemented soil to that of the corresponding reconstituted soil. In Figure 5(a) adapted from Cotecchia and Chandler (2000), the position of the SBS of the cemented soil is defined with parameters  $N$  and  $\lambda^*$ . The current SBS relates to  $p_e'$ , that is the Hvorslev equivalent pressure defined as

$$p_e' = \exp\left(\frac{N - \ln(1+e)}{\lambda^*}\right) \quad (1)$$

where  $\lambda^*$  is the slope of the NCL,  $N$  is the value of  $\ln(1+e)$  at  $p'=p_r= 1 \text{ kPa}$  and  $e$  is void ratio. For the reconstituted soil the position of the SBS is defined with parameters  $N^*$  and  $\lambda^*$  (Figure 5(a)), which determine the Hvorslev equivalent pressure  $p_e^{*}$  for the reconstituted soil. The current state of the structure can be quantified by the ratio  $s = p_e'/p_e^{*}$ .

As indicated in Figure 5(a), the value of  $N^*$  is constant, whereas  $N$  decreases due to structure degradation in compression after yield (SBS plotted in Figure 5(a) corresponds to the undisturbed state of cemented soil). Consequently, the value of  $s$  is constant until the yield stress, and decreases with progressive debonding after yield. Following Baudet and Stallebrass (2004), it is assumed that a stable component of structure may exist, which does not degrade in continuous compression and shear. Therefore, sensitivity  $s$  will decrease until its final value ( $s_f$ ), which represents the stable elements of structure.

#### VARIATION OF THE VERY SMALL STRAIN SHEAR MODULUS

For reconstituted soil the influence of the effective stress state and stress history on  $G_0$  may be expressed by the relationship (Viggiani and Atkinson 1995)

$$\frac{G_0}{p_r} = A \left( \frac{p'}{p_r} \right)^n \left( \frac{p_{p'}}{p'} \right)^m \quad (2)$$

where  $p_r$  is reference pressure (1 kPa) and  $p_{p'}$  is the yield stress. Thus the ratio  $p_{p'}/p'$  defines the isotropic overconsolidation ratio.  $A$ ,  $n$  and  $m$  are dimensionless soil parameters.

To represent the experimental data obtained for the cemented soils, equation (2) can be modified to the form including the effect of structure

$$\frac{G_0}{p_r} = A \left( \frac{p'}{p_r} \right)^n \left( \frac{p_{e'}}{p'} \right)^m \left( \frac{s}{s_f} \right)^l \quad (3)$$

where  $l$  is a new parameter controlling the influence of soil structure on  $G_0$ . The equation is equivalent to that proposed by Cafaro and Cotecchia (2001) for natural clays including a diagenized bonding, who explained how this equation results from the conceptual framework reported by Cotecchia and Chandler (2000), based on the selection of the parameter  $s$  to represent the comparison between the strength of the natural clay structure and that of the reconstituted clay structure. For the reconstituted soil the values are  $s=s_f=1$  and equation (3) is reduced to equation (2) expressed in terms of the Hvorslev equivalent pressure ( $p_{e'}$ ).

Equation (3) may be rewritten as

$$\ln \left( \frac{G_0}{p_r} \right) = \ln A + n \ln \left( \frac{p'}{p_r} \right) + m \ln \left( \frac{p_{e'}}{p'} \right) + l \ln \left( \frac{s}{s_f} \right) \quad (4)$$

According to equation (4) and Figure 5(b), the value of  $A$  represents  $G_0$  of the reconstituted soil at  $p'=p_r=1$ , the parameter  $n$  relates  $G_0$  to  $p'$  and the parameter  $m$  specifies the effect of the overconsolidation ratio (defined as  $p_e^*/p'$ ). For cemented soil,  $G_0$  is increased at the pre-yield state by the apparent overconsolidation ratio (defined as  $p_e'/p'$ ), which is higher than for the corresponding reconstituted soil and is accounted for in the third term of the sum in equation (4), and by the sensitivity ( $s$ ). After yield, the effect of the apparent overconsolidation ratio disappears ( $p_e'=p'$ ) and the effect of sensitivity prevails.

Equation (3) was used to calculate  $G_0$  for the three studied soils. The values of the parameters are summarised in Table 1. The same values of  $A$ ,  $n$  and  $m$  were used for the corresponding cemented and reconstituted soils. The parameter  $l$  was calibrated by fitting a curve through the experimental data for the structured soil. In the case of Nevada sand the same value of parameter  $l$  was used for both amounts of Portland cement. For calcarenite and kaolin clay  $s_f=1$  was considered due to converging NCLs of the cemented and reconstituted specimens (Figures 1(b) and 3(a)). For Nevada sand  $s_f=35$  was chosen so that it represents the large distance and slow convergence of the NCLs (Figure 4(a)).

The measured and calculated data (equation (3)) are compared in Figures 2, 3(b) and 4(b), showing a good fit in the three cases. The  $G_0$  calculated for the cemented soils decreases slightly before yielding as  $p'$  increases and  $p_e'/p'$  decreases at the same time. The equation thus cannot predict constant  $G_0$  pre-yield, measured for strongly cemented soils. The  $G_0$  variation with  $p'$  is, however, minor due to the low pre-yield compressibility of cemented soils and the measurements are relatively well represented. In Figures 3(b) and 4(b) the irregularities in the decreasing trend of calculated  $G_0$  are attributed to the scatter in the experimental values of  $p_e'$  used in equation (3).

#### QUALITATIVE PREDICTION OF $G_0$ FOR DIFFERENT RATES OF STRUCTURE DEGRADATION

According to the Introduction, for cemented soils either reduction or increase of  $G_0$  after yielding was reported. To explain this phenomenon, various isotropic compression curves (Figure 6(a)) were simulated by the model for structured clays (Mašín 2007). Behaviour of cemented soils in the post-yield stress regime depends significantly on the current state of structure. The model enables to control the rate of structure degradation by a parameter  $k$ . The current value of  $s$  may be expressed by the relationship

$$s = s_f + (s_0 - s_f) \left[ -\frac{k}{\lambda} \varepsilon^d \right] \quad (5)$$

where  $s_0$  is initial sensitivity,  $k$  is the rate of structure degradation,  $\lambda^*$  is the gradient of NCL of reconstituted soil and  $\varepsilon^d$  is damage strain.

Figure 6(b) shows the development of  $G_0$  calculated from the simulated data using equation (3). The results indicate that after yield  $G_0$  increases with  $p'$  at low rates of structure degradation ( $k = 0.3$ ) and drops at high rates ( $k = 1$ ).

## CONCLUSIONS

The data shows that the large-strain compression behaviour of a mixture of kaolin clay and 4% of Portland cement is comparable to the behaviour of natural clays. Bender element measurements indicate a significant influence of cementation on  $G_0$ . The agreement between the test results and the equation, relating  $G_0$  to the mean stress, apparent overconsolidation ratio and sensitivity, proposed by Cafaro and Cotecchia (2001) for natural clays, is obtained. Moreover, the applicability of the equation to cemented sands is shown. After yield, the development of  $G_0$  is interpreted with the rate of structure degradation.

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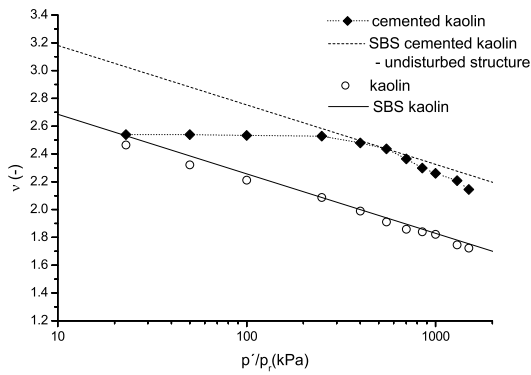
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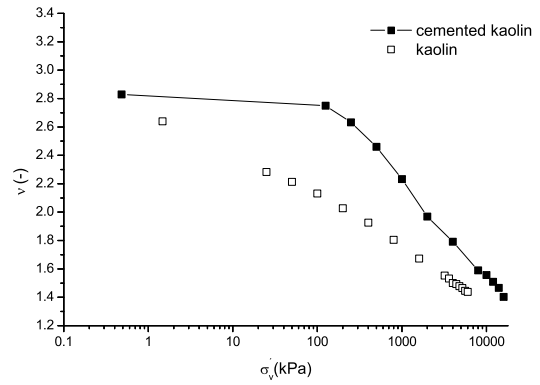
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Table 1. Parameters controlling the shear modulus of soils with cementation bonds used in equation (3)

material:	A	n	m	l	$s_r$
kaolin clay	1020	0.73	0.77	0.24	1
calcarenite	2326	0.631	0.7	0.7	1
Nevada sand	2454	0.642	0.7	0.34	35



(a)



(b)

Figure 1: Compression lines of pure and cemented kaolin clay: (a) isotropic compression; (b) oedometer compression after 14 days of curing

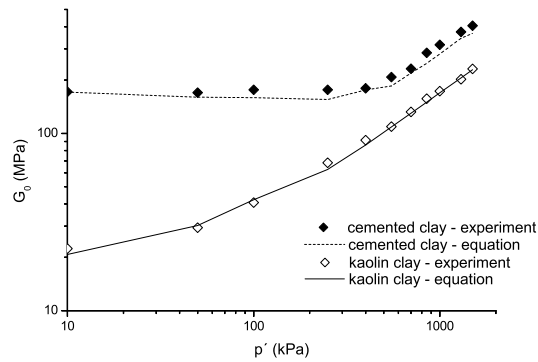


Figure 2: Very small-strain shear modulus plotted against mean effective stress measured on specimens of cemented and pure kaolin clay using bender elements and obtained from equation (3)

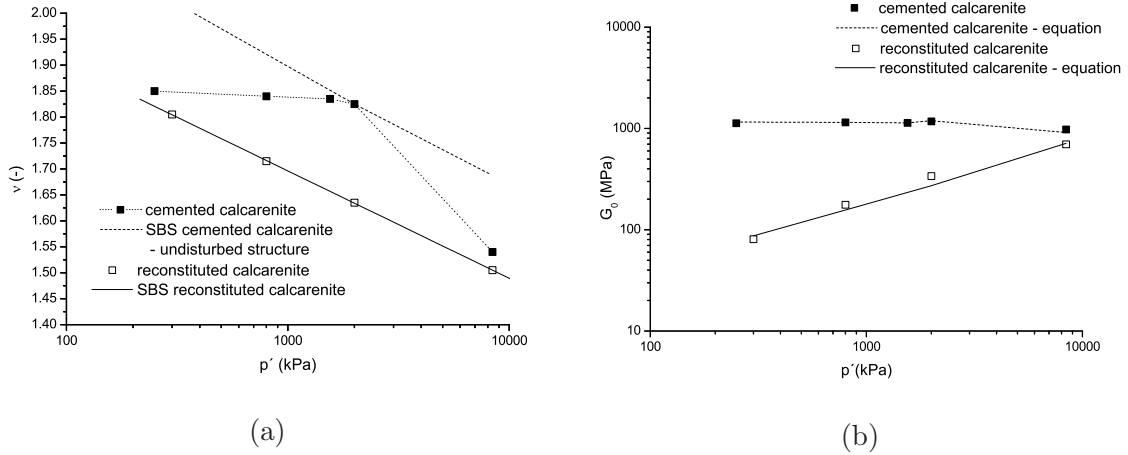


Figure 3: Intact and reconstituted calcarenite (experimental data from Cuccovillo & Coop (1997)): (a) isotropic compression lines; (b) very small-strain shear modulus plotted against mean effective stress measured using LVDTs and obtained from equation (3)

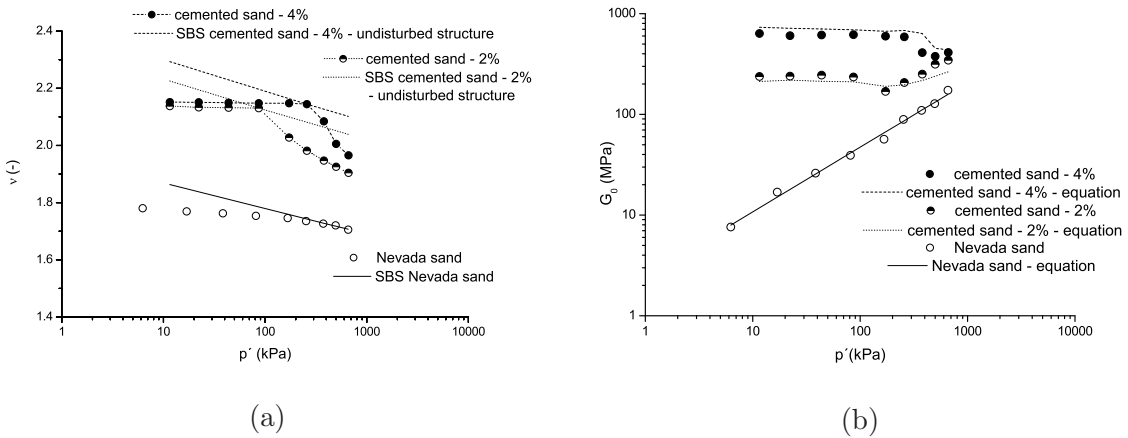


Figure 4: Cemented and pure Nevada sand (experimental data from Yun & Santamarina (2005)): (a) oedometer compression lines; (b) comparison of results for very small-strain shear modulus obtained from equation (3) and from bender element measurements

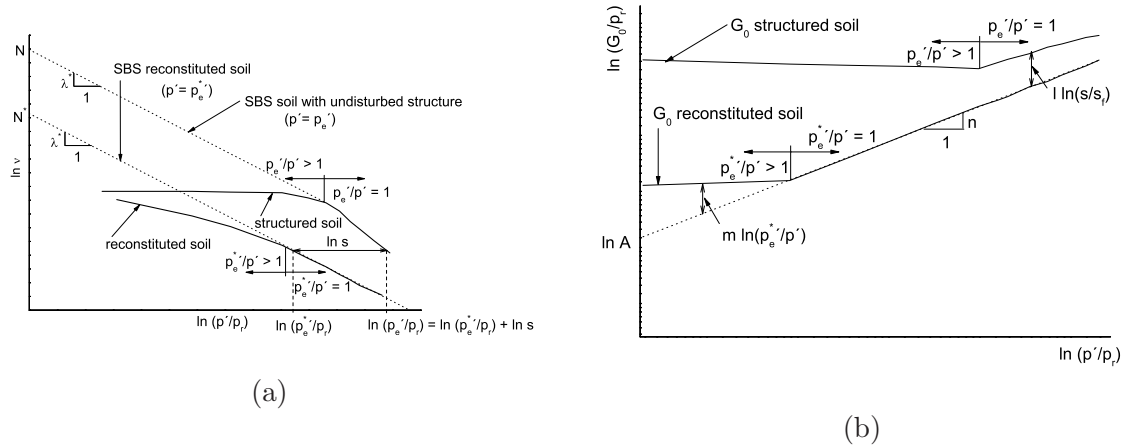


Figure 5: Schematic diagrams showing the behaviour of reconstituted and structured soil with the definition of variables: (a) compression (revisited from Cotecchia & Chandler (2000)); (b) shear modulus at very small strain

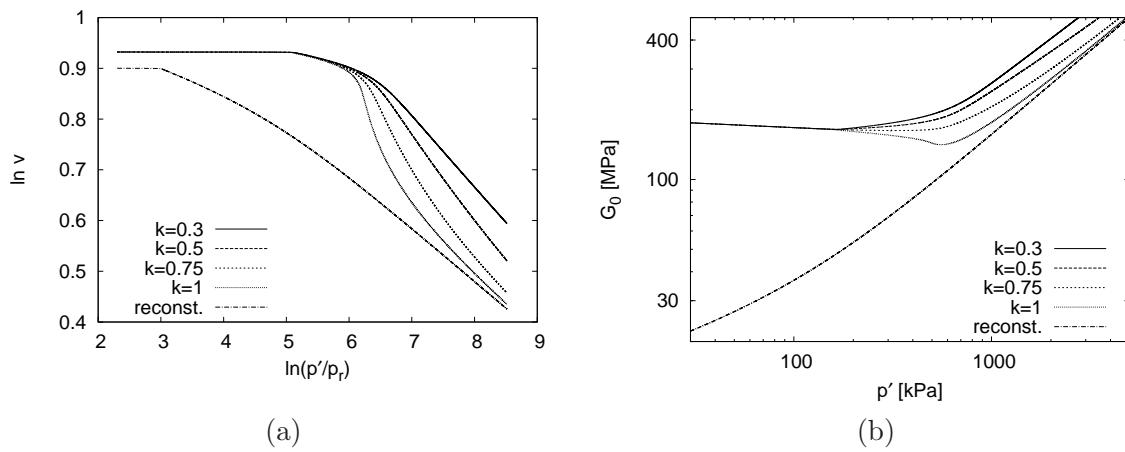


Figure 6: Effect of the rate of structure degradation  $k$  on the mechanical behaviour of cemented soil: (a) isotropic compression tests; (b) variation of  $G_0$  with  $p'$