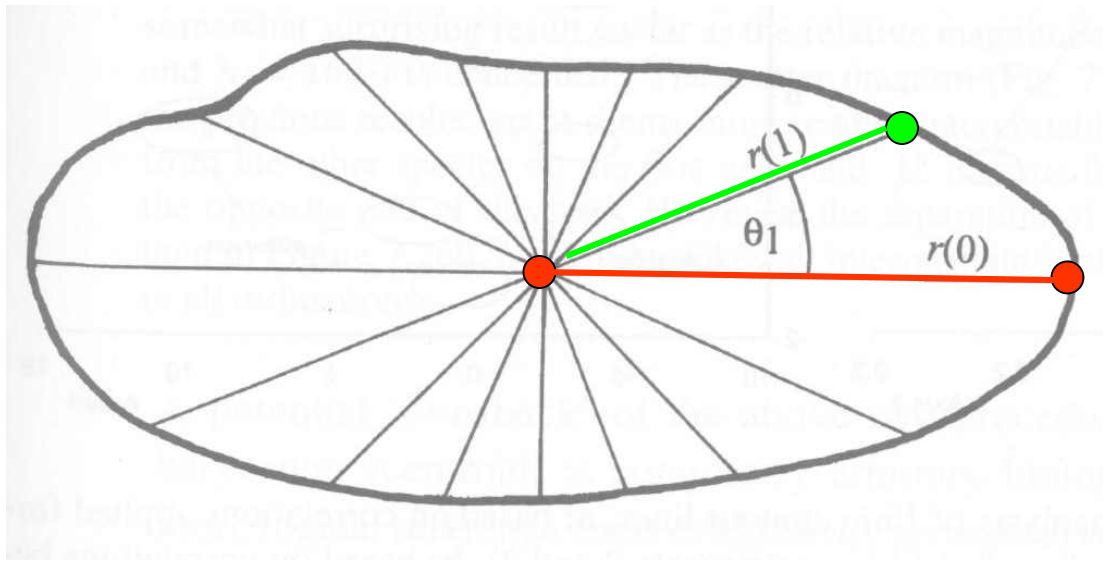


Outline analysis: Eigenshape analysis

Radius function $r(\theta)$

The reference line is a radius drawn from the centroid (or another corresponding point) to a primary landmark. From this radius, additional lines are drawn at equal angles. The radial function describes the relationship between the angle of rotation and the corresponding radius length.

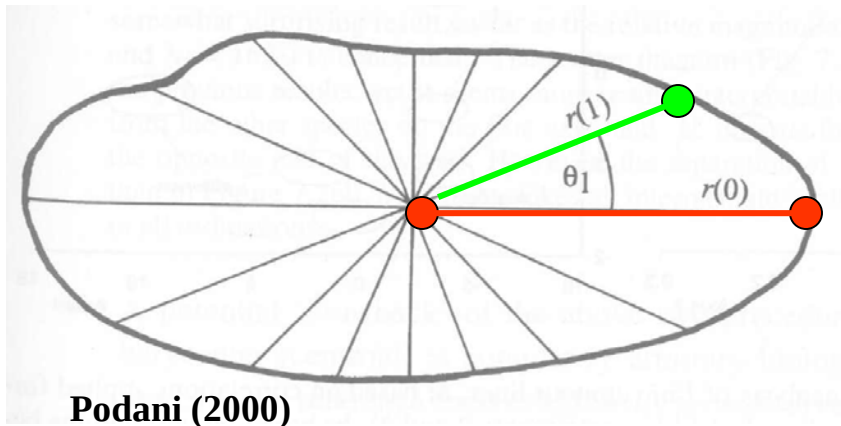


Outline analysis: Eigenshape analysis

Radius function $r(\theta)$

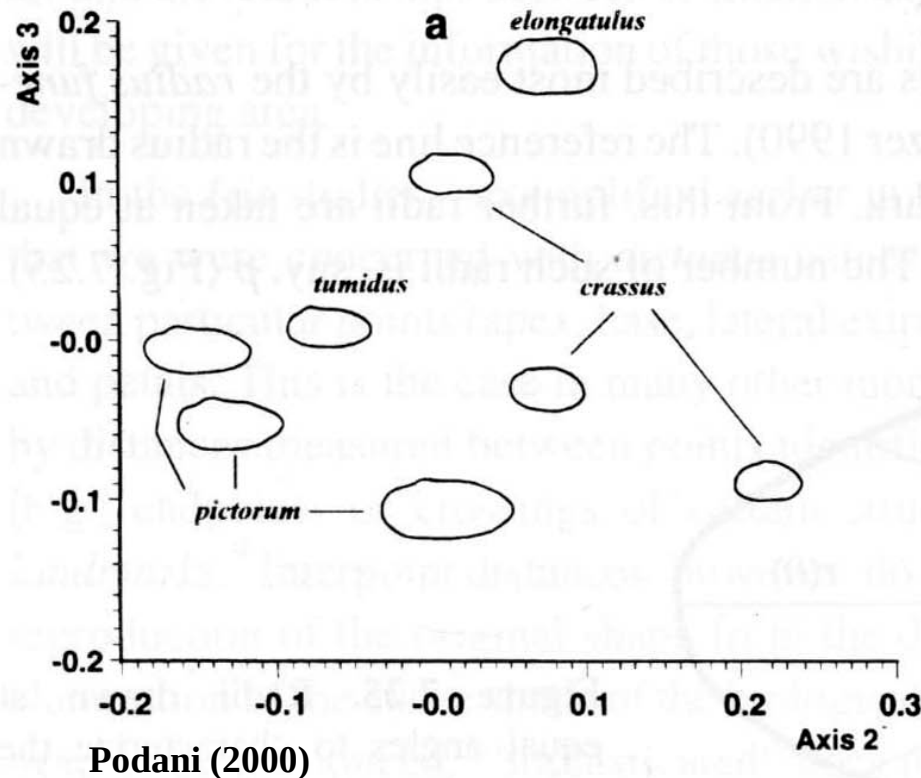
If the number of radii drawn from the centroid is sufficient (p), their lengths describe the studied shape quite well.

The result is a $p \times m$ matrix, where m is the number of studied objects. The matrix forms the basis for PCA, which is based on correlations between objects (not between features) – eigenshape analysis.

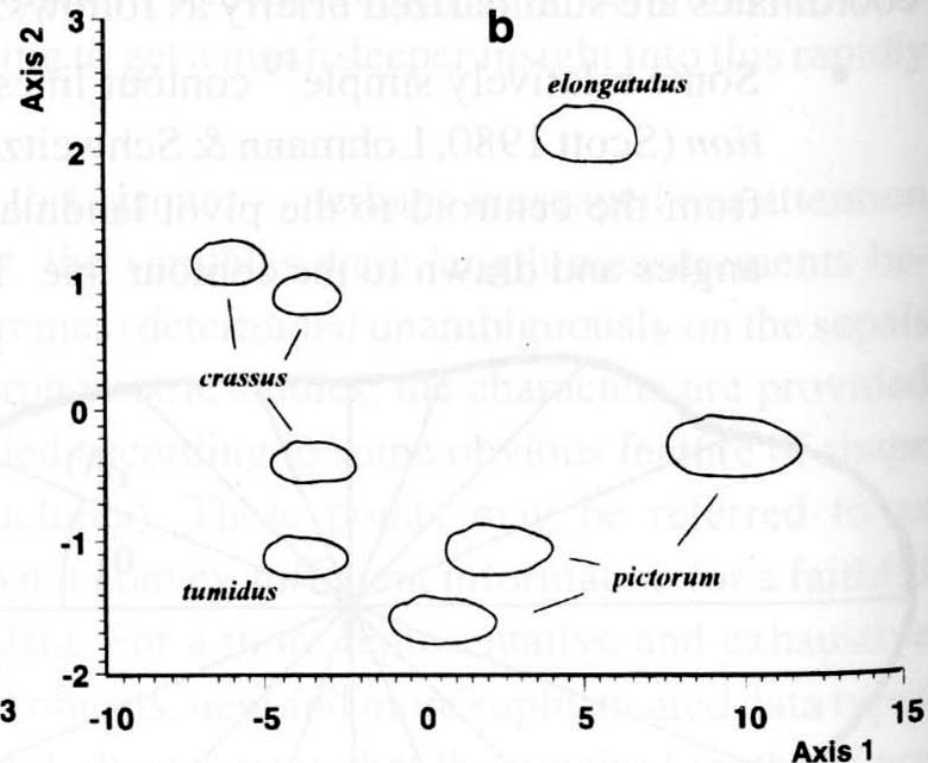


Eigenshape analysis - Radius function $r(\theta)$

Unio PCA – eigenshape analysis, the first eigenvalue accounts for 97% – size component, so it is better to assess the second and third axes afterwards.



Unio PCA – correlations of traits, the first eigenvalue accounts for 91.3%, the second for 5.2%.

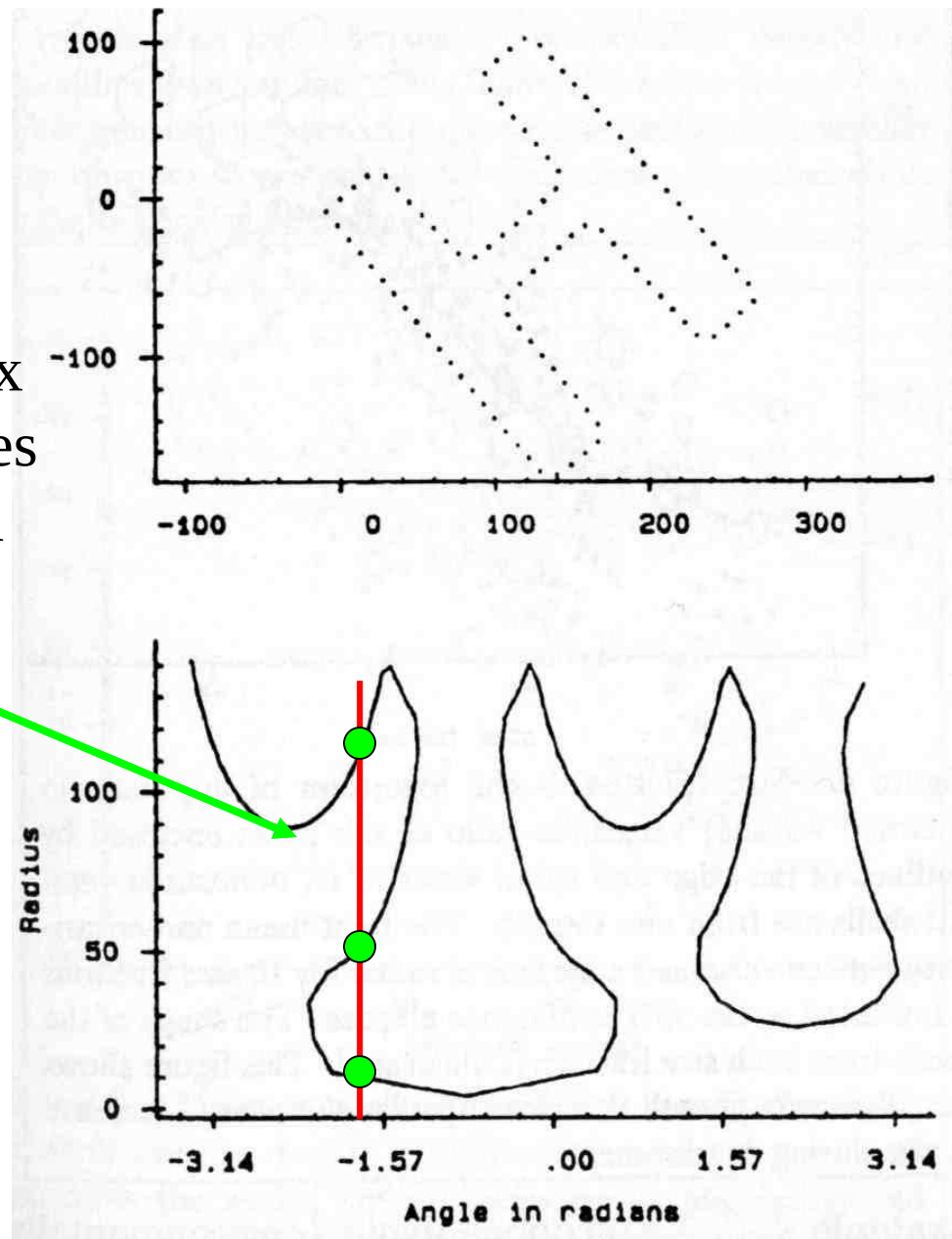


Eigenshape analysis

Radius function – $r(\theta)$

For more complex shapes, this function is not suitable for describing more complex forms – in some cases, r does not have a single value for a given angle θ .

The choice of the central landmark or centroid is also problematic – if its position is variable in relation to the biological structure, it significantly influences the result.



Eigenshape analysis

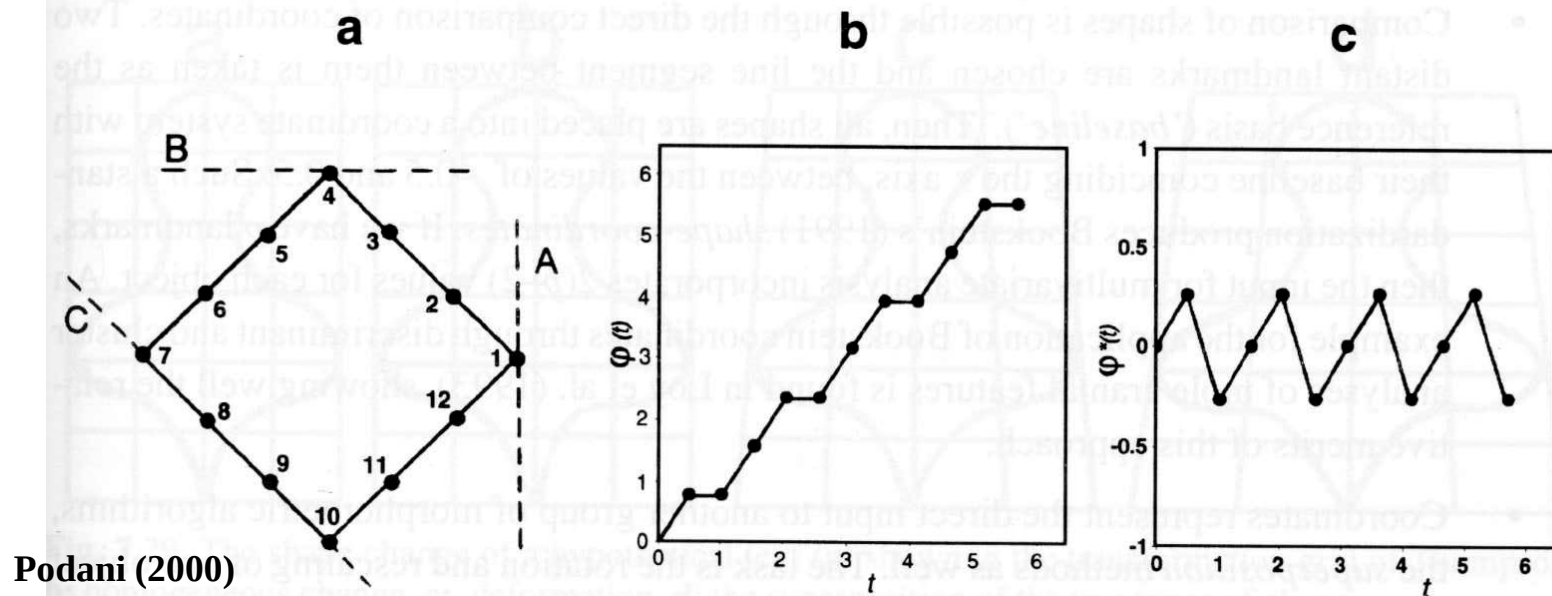
Zahn & Roskies' shape function

$$\varphi^*(t) = \varphi(t) - t$$

t is the distance from the initial (zero) reference point along the outline, normalized to the total length $2\pi r$.

$\varphi(t)$ is the angle in radians between the tangential vector at point 0 and the vector at a distance t from the zero point.

The function has a value of zero for a circular outline.



Eigenshape analysis

Zahn & Roskies' shape function

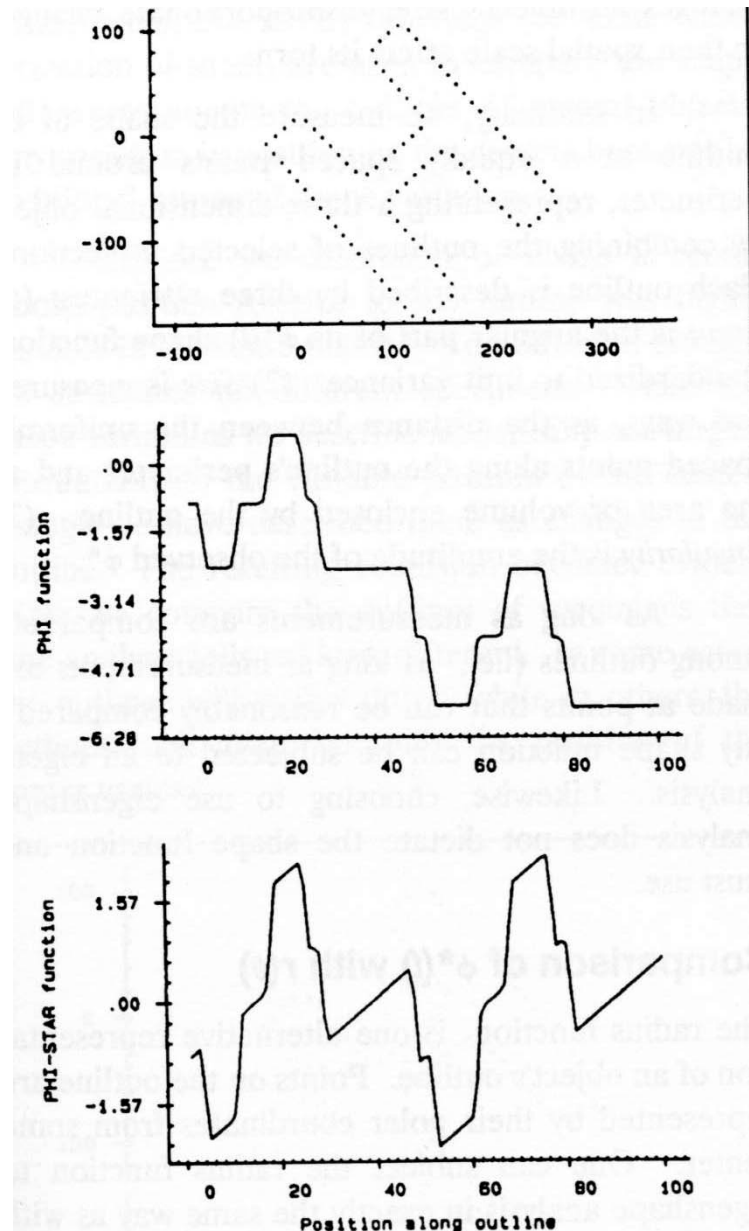
The result is a matrix that is used for the analysis of principal shapes (PCA). "Sampling" of the outline is therefore not at equal angles but at equal distances along the outline. The shorter these distances, the more accurate the result (minimum $n = 100$).

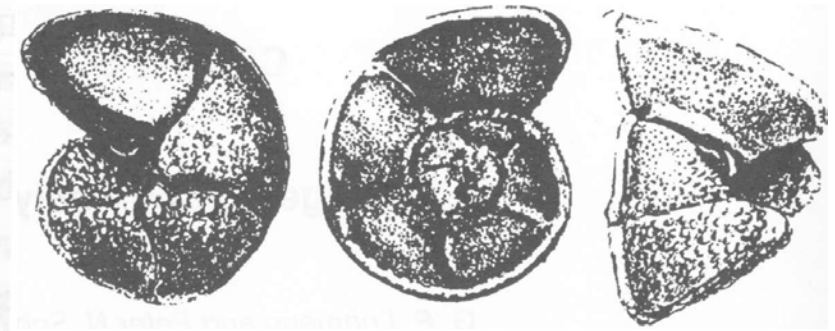
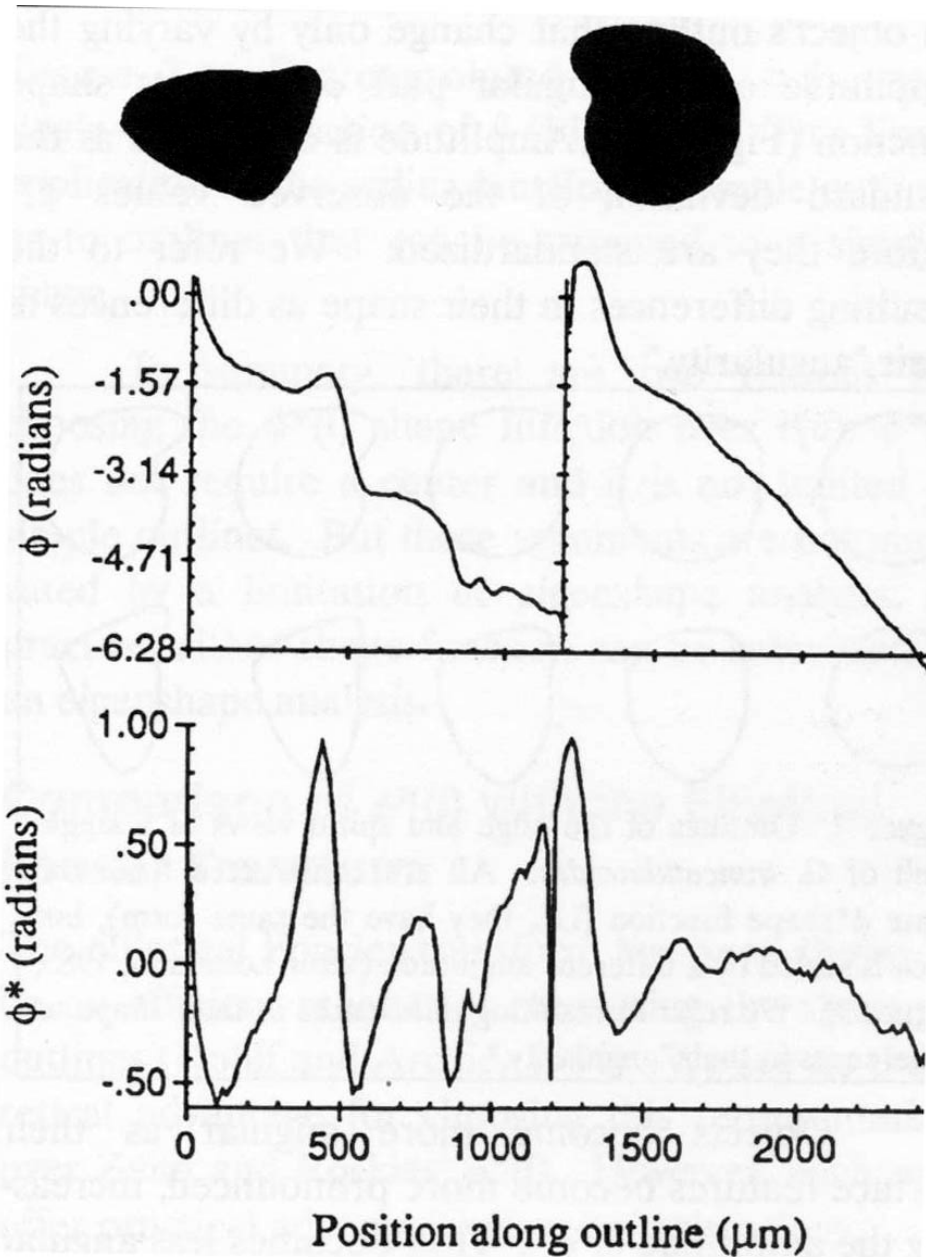
Shape – the function $\varphi^*(t)$

Size – $S = t / n$

Angularity – amplitude

$$\varphi^*(t) = \varphi(t) - t$$





Globorotalia truncatulinoides

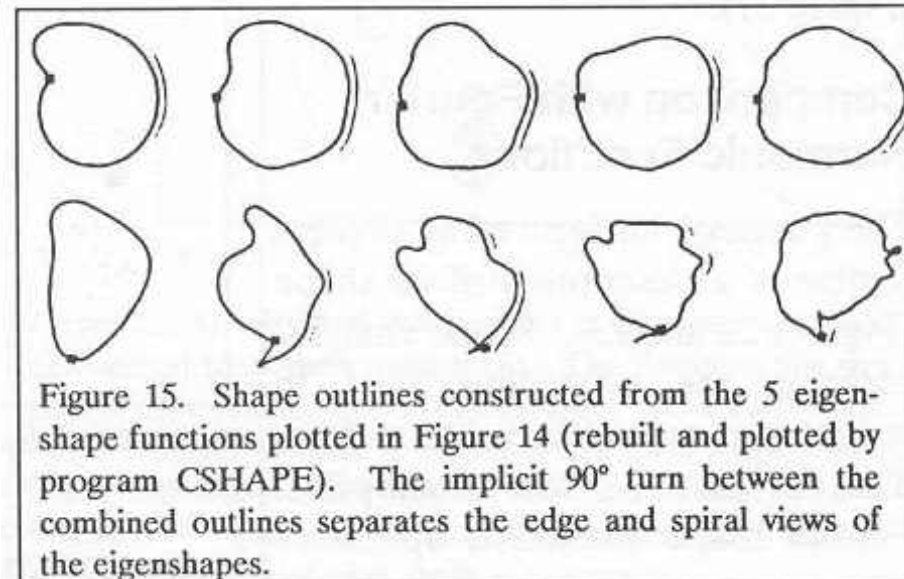
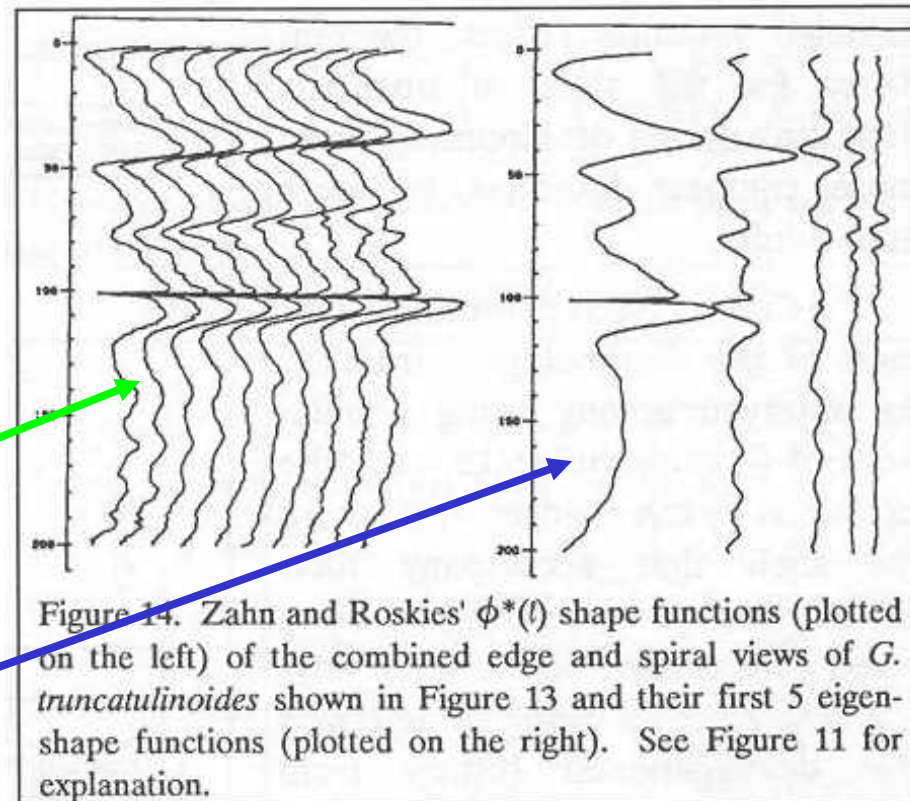
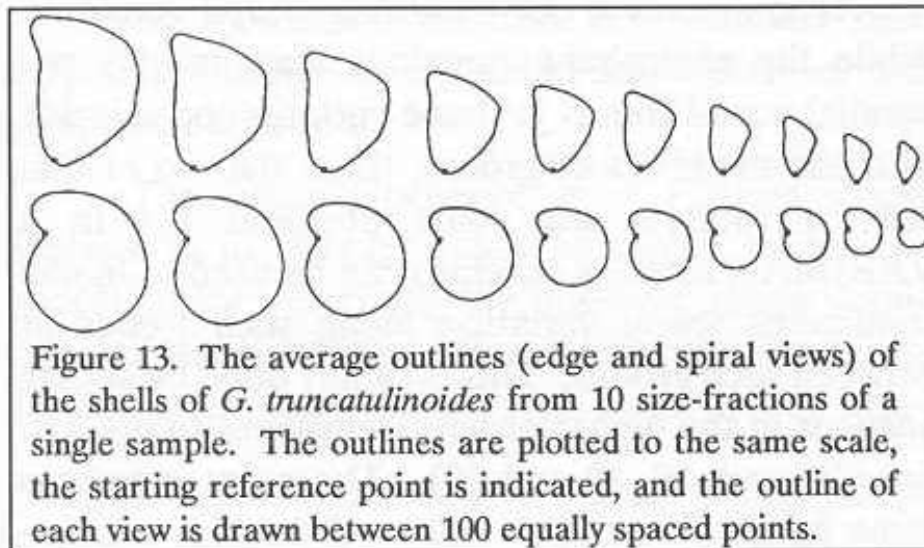
The initial reference point and the direction of data collection along the outline are important!
(in the direction or opposite direction of the clock hands)

Eigenshape analysis

Zahn & Roskies' shape function

Shape function

Eigenshape function



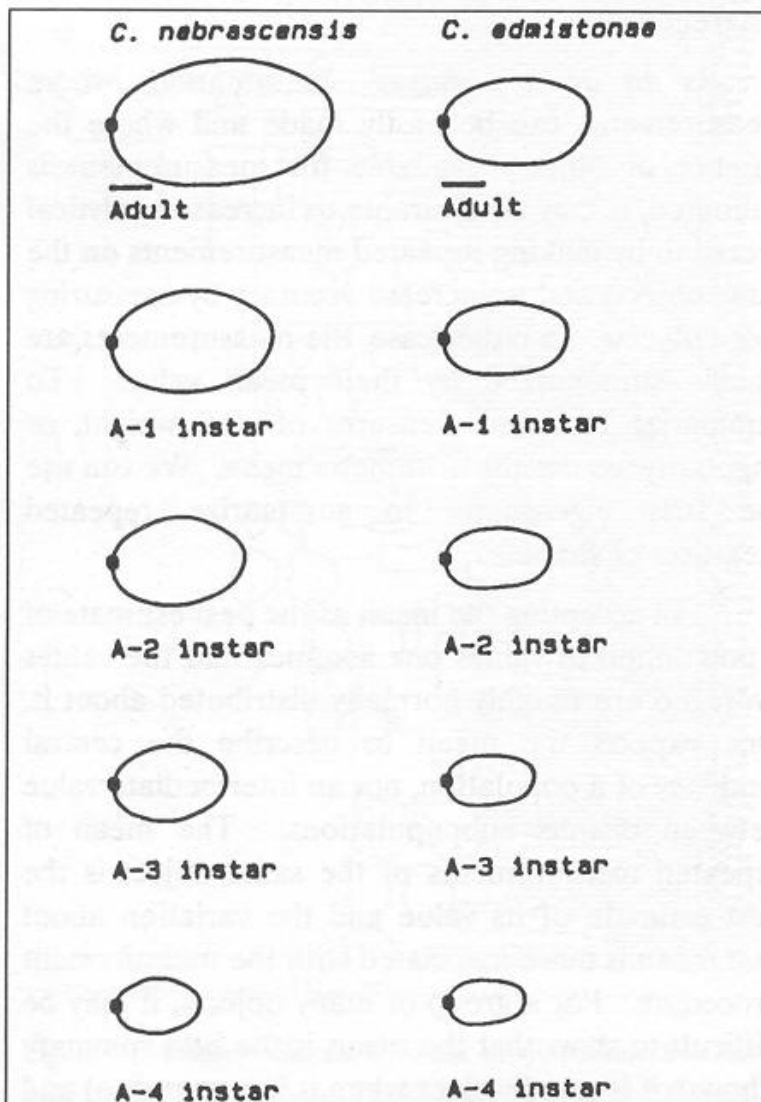


Figure 10. The average outlines of ostracode shells from 5 instars (growth stages) in each of two related species of *Cavellina* (from Schweitzer et al., 1986). The outlines are plotted to the same scale, the starting reference point is indicated, and each outline is drawn between 128 equally spaced points.

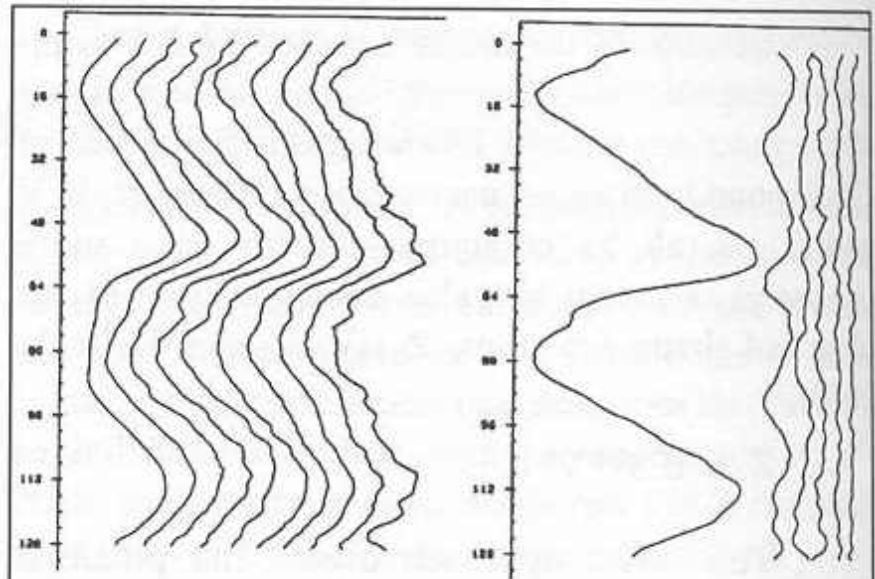


Figure 11. Zahn and Roskies' $\phi^*(l)$ shape functions (on the left) of the ostracode outlines shown in Figure 10 and their first 5 eigenshape functions (on the right). The values of ϕ^* , the net angular bend around each outline, are plotted on the horizontal axis. The vertical axis orders the 128 comparable points located around the perimeter of each outline.

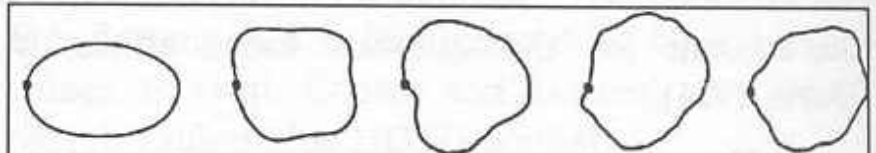
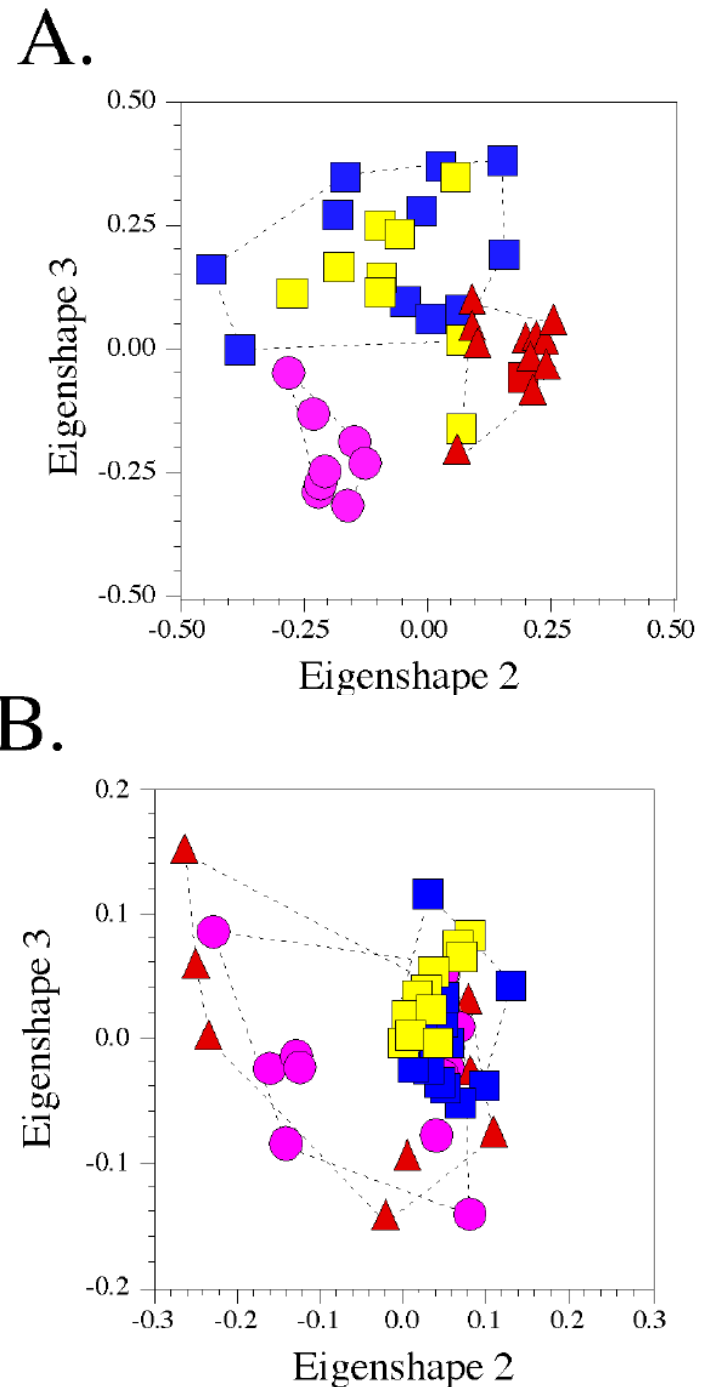


Figure 12. Shape outlines constructed from the 15 eigenshape functions plotted in Figure 11 (rebuilt and plotted by program CSHAPE). Only the first eigenshape resembles the ostracodes that were analyzed. Since it accounts for most of the "variance" among a collection of similar objects, it describes features they all share, features that make them look the same, and it approximates the mean shape. The other eigenshapes account for the remaining "variance" contributed by features that make the objects look different.

The role of phylogeny in quantitative paleobiological data analysis

Norman MacLeod

Figure 5. Results of MacLeod and Rose's (1993) shape analysis of modern mammalian distal phalanx (A) and proximal radial head (B) data. Symbols as follows: solid square = arboreal species, open squares = scansorial species, solid triangles = fossorial species, solid circles = cursorial species (see MacLeod and Rose 1993 Appendix 1 for species lists). Note that the ordination of species on the two most important distal phalanx shape-dissimilarity axes (eigenshape axes 2 and 3) separate the data set into three locomotor groups (with a small number of intergroup outliers) whereas the group separation in the proximal radial head analysis is much less pronounced. Since each locomotor group contains species from different mammalian clades, a functional signal may be overwhelmed by the phylogenetic signal in the latter analysis

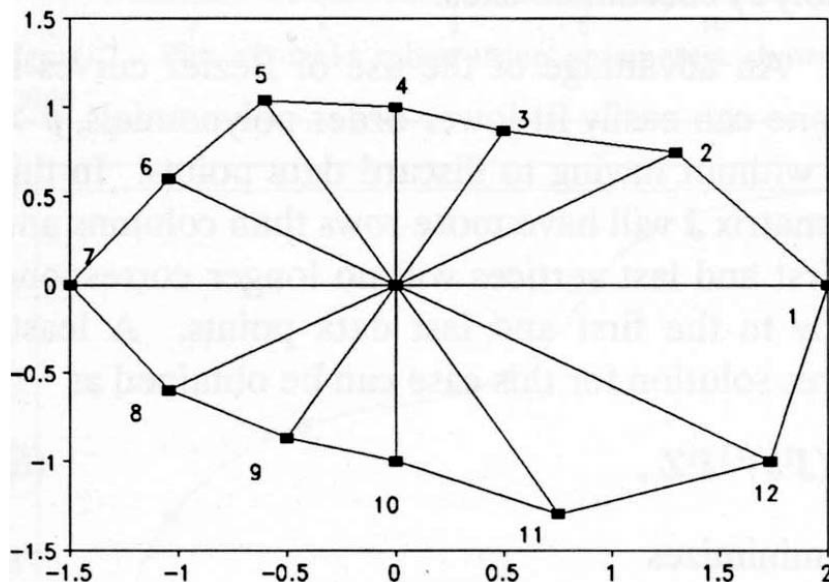


Outline analysis: Fourier analysis

Fourier analysis using equally spaced radii

$$\theta = a_0 + \sum_{i=1}^k (a_i \cos(i \theta) + b_i \sin(i \theta))$$

The angle θ varies from **0 to 2π** , a_i b_i – Fourier coefficients of the i -th harmonic function, k – the maximum number of harmonic functions:
 $k < p / 2$.



$$a_i = \sqrt{\frac{2}{p}} \sum_{j=1}^p \theta_j \cos(i \theta_j)$$

$$b_i = \sqrt{\frac{2}{p}} \sum_{j=1}^p \theta_j \sin(i \theta_j)$$

$$a_0 = \sqrt{\frac{1}{p}} \sum_{j=1}^p \theta_j$$

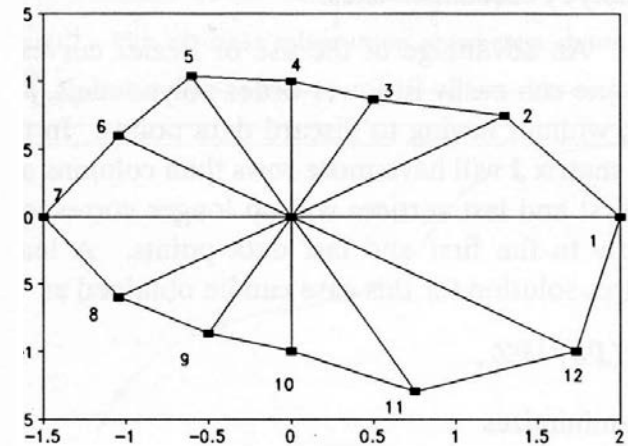
$$b_0 = 0 .$$

Outline analysis: Fourier analysis

Fourier analysis using equally spaced radii

Table 7: Lengths, ρ , of radii at an angle θ and Fourier coefficients for harmonics 0 through 6.

θ	ρ	i	a_i	b_i
0.00000	2.0	0	1.34167	0.00000
0.52360	1.5	1	0.65427	-0.20813
1.04720	1.0	2	0.85732	-0.42426
1.57080	1.0	3	0.08165	-0.20412
2.09440	1.2	4	0.08165	0.07071
2.61799	1.2	5	-0.12355	0.00400
3.14159	1.5	6	0.12247	0.00000
3.66519	1.2			
4.18879	1.0			
4.71239	1.0			
5.23599	1.5			
5.75959	2.0			
6.28319	2.0			



Fourier analysis

Fourierova analysis of tangent angles

$$\phi^*(t) = \phi(t) - t$$

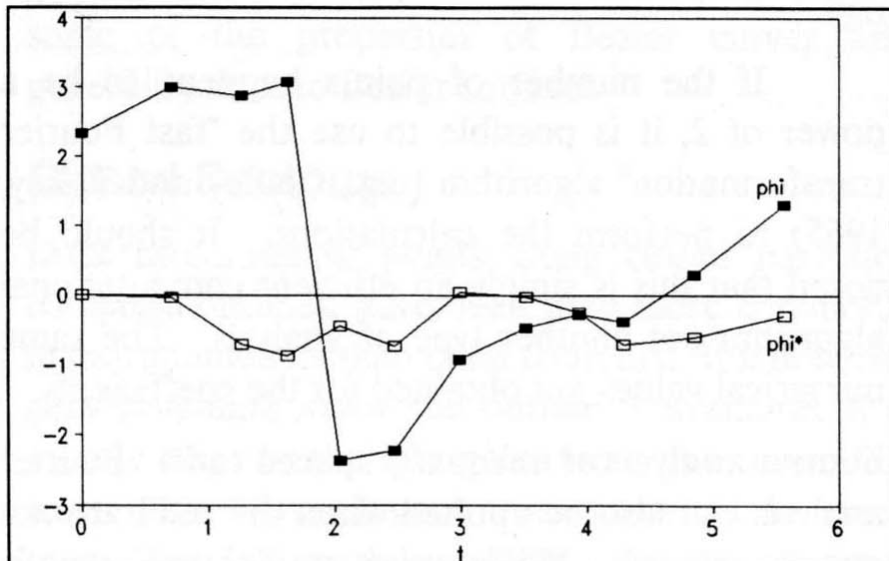


Figure 6. Plot of ϕ and ϕ^* as functions of cumulative chordal distance t .

Table 8. Zahn and Roskies (1972) ϕ^* values and their Fourier coefficients for the example data of Table 7.

t	ϕ	ϕ^*	i	a_i	b_i
0	2.32241	0	0	-0.34626	
0.71074	2.99739	-0.03575	1	0.17595	-0.10944
1.26974	2.87979	-0.71236	2	0.91564	0.26300
1.62813	3.07630	-0.87424	3	-0.15540	0.02055
2.04442	-2.35619	-0.43984	4	-0.02769	0.37984
2.47448	-2.22569	-0.73939			
2.99824	-0.91591	0.04663			
3.52200	-0.45831	-0.01954			
3.93829	-0.26180	-0.23932			
4.29668	-0.37940	-0.71530			
4.85568	0.29558	-0.59933			
5.56642	1.30900	-0.29665			

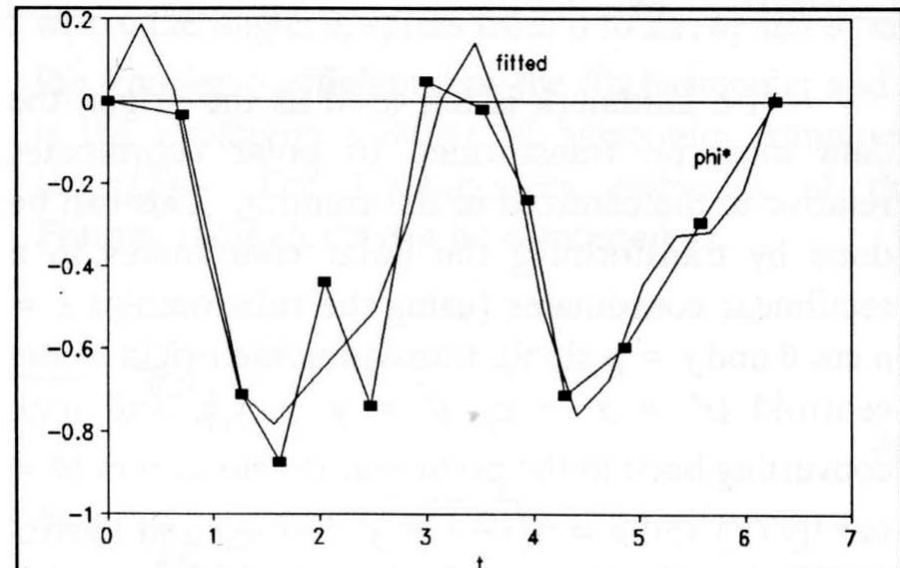
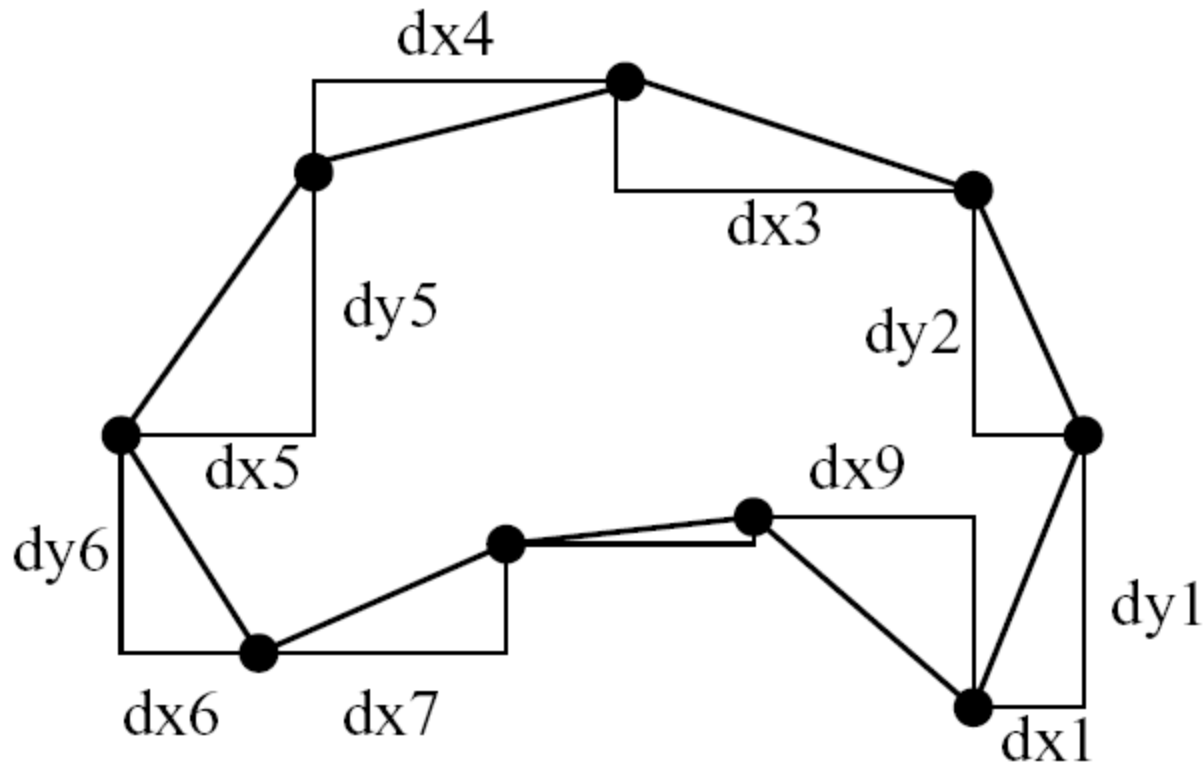


Figure 7. Plot of ϕ^* and an estimate of ϕ^* based on the first four harmonics.

Fourier analysis

Elliptic Fourier analysis

The positions of points and steps on the outline are characterized by x and y coordinates.



Fourier analysis

Elliptic Fourier analysis

The function we are looking for describes the simultaneous change of coordinates, i.e., Δx and Δy in accordance with a set of harmonic functions. For each harmonic function, 4 coefficients are derived (two for horizontal coordinates and two for vertical coordinates). In addition, two constants are calculated.

Fourier coefficients for the k -th harmonic function and the x -projection of the outline are calculated as follows:

$$A_k = \frac{T}{2p^2\pi^2} \sum_{i=1}^p \frac{\Delta x_i}{\Delta t_i} \left[\cos \frac{2\pi k t_i}{T} - \cos \frac{2\pi k t_{i-1}}{T} \right]$$
$$B_k = \frac{T}{2p^2\pi^2} \sum_{i=1}^p \frac{\Delta x_i}{\Delta t_i} \left[\sin \frac{2\pi k t_i}{T} - \sin \frac{2\pi k t_{i-1}}{T} \right]$$

$\Delta x_i = x_i - x_{i-1}$ – chordal distance of the step between points i and $i - 1$
 t_i – cumulative length of steps up to step i
 T – total length of the outline

Rohlf (1990, „Blue Book“)

The constant for the x-coordinates is calculated as follows:

$$A_0 = x_0 +$$

$$\frac{1}{T} \sum_{i=1}^p \left[\frac{\Delta x_i}{2\Delta t_i} (t_i^2 - t_{i-1}^2) + \left[\sum_{j=1}^{i-1} \Delta x_j - \frac{\Delta x_i}{\Delta t_i} t_{i-1} \right] \Delta t_i \right]$$

The Fourier coefficients for the k -th harmonic function and the y-projection of the outline are calculated equivalently.

For the entire curve, n harmonic functions, and t from the interval $0 - 2\pi$, the following holds:

$$x(t) = A_0 + \sum_{k=1}^n A_k \cos kt_k + B_k \sin kt_{k-1}$$

$$y(t) = C_0 + \sum_{k=1}^n C_k \cos kt_k + D_k \sin kt_{k-1}$$

Fourier coefficients can then be used for further analyses – PCA, cluster analyses, discriminant analyses, etc.

Fourier analysis

Elliptic Fourier analysis

θ	ρ
0.00000	2.0
0.52360	1.5
1.04720	1.0
1.57080	1.0
2.09440	1.2
2.61799	1.2
3.14159	1.5
3.66519	1.2
4.18879	1.0
4.71239	1.0
5.23599	1.5
5.75959	2.0
6.28319	2.0

Table 9. Elliptic Fourier coefficients for the example data of Table 7.

i	A_i	B_i	C_i	D_i
0	0.31460	—	-0.09995	—
1	1.56468	-0.27246	0.00959	1.13297
2	-0.01796	0.00444	0.00135	0.03388
3	0.06718	-0.03061	0.08219	0.03999

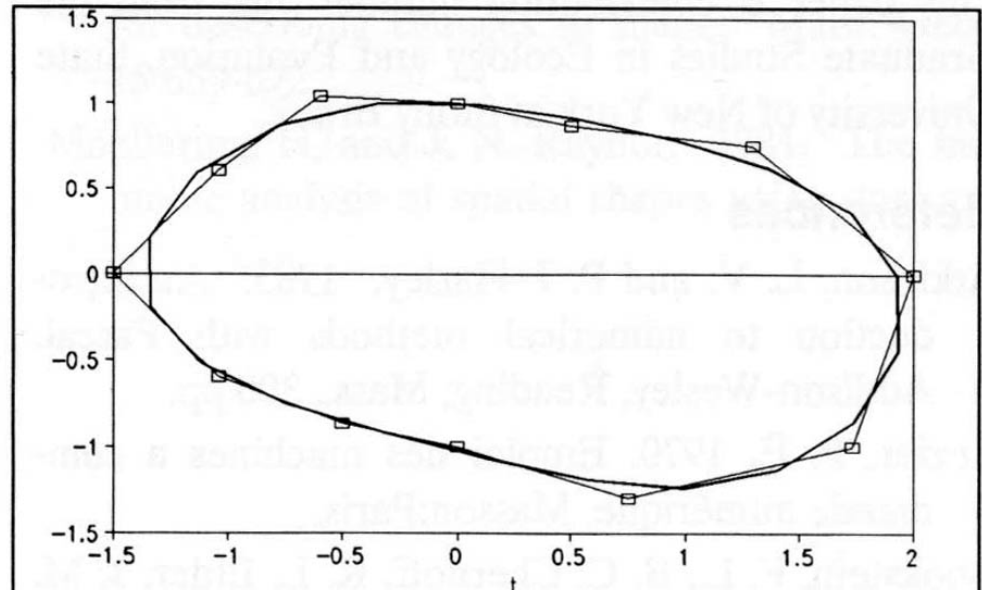
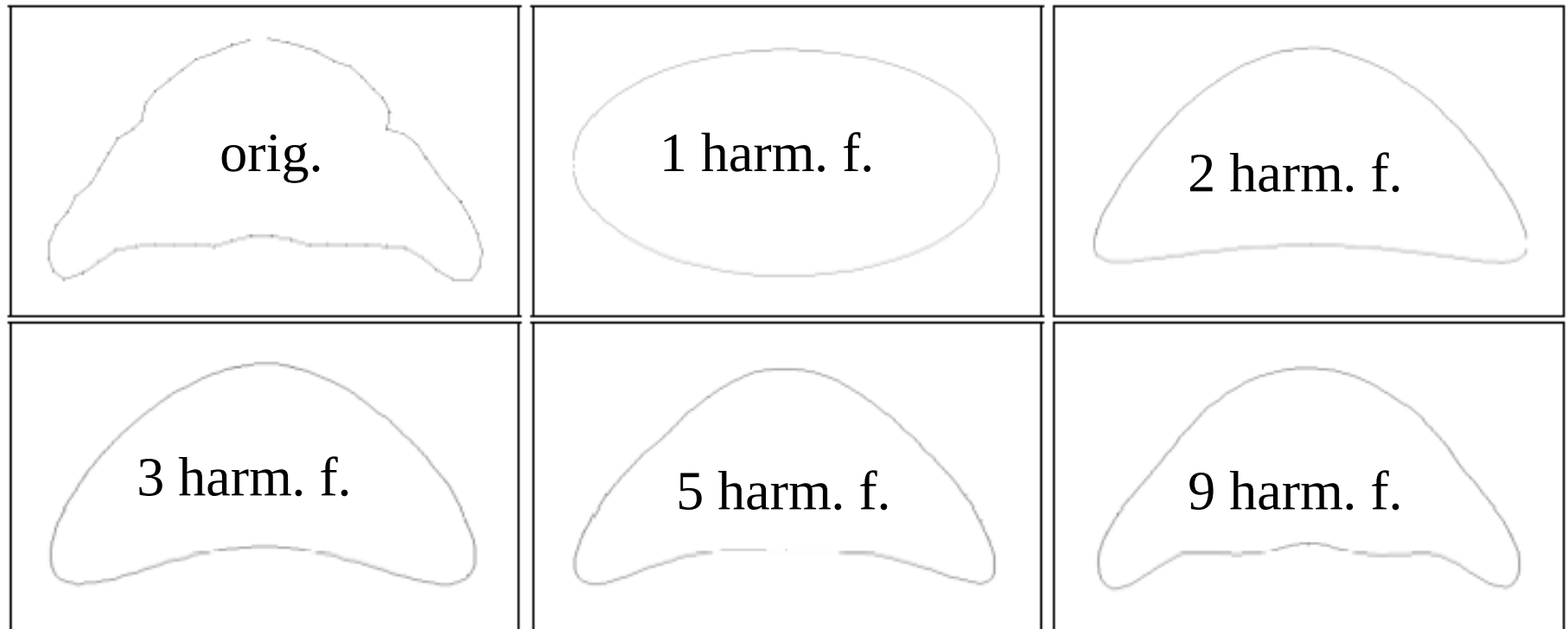
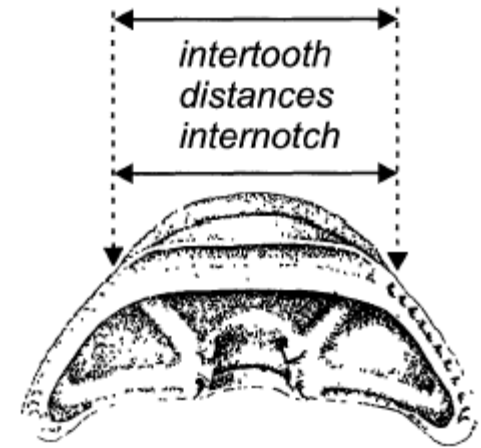


Figure 8. Plot of elliptic Fourier function based on the first 3 harmonics for the example data (squares connected with thin lines) of Table 7.

**Ontogeny of *Trimerocephalus lelievrei* (Trilobita, Phacopida),
a representative of the Late Devonian phacopine
paedomorphocline: a morphometric approach**

Catherine Crônier, Sabrina Renaud, Raimund Feist, and Jean-Christophe Auffray

64 points characterizing the outline



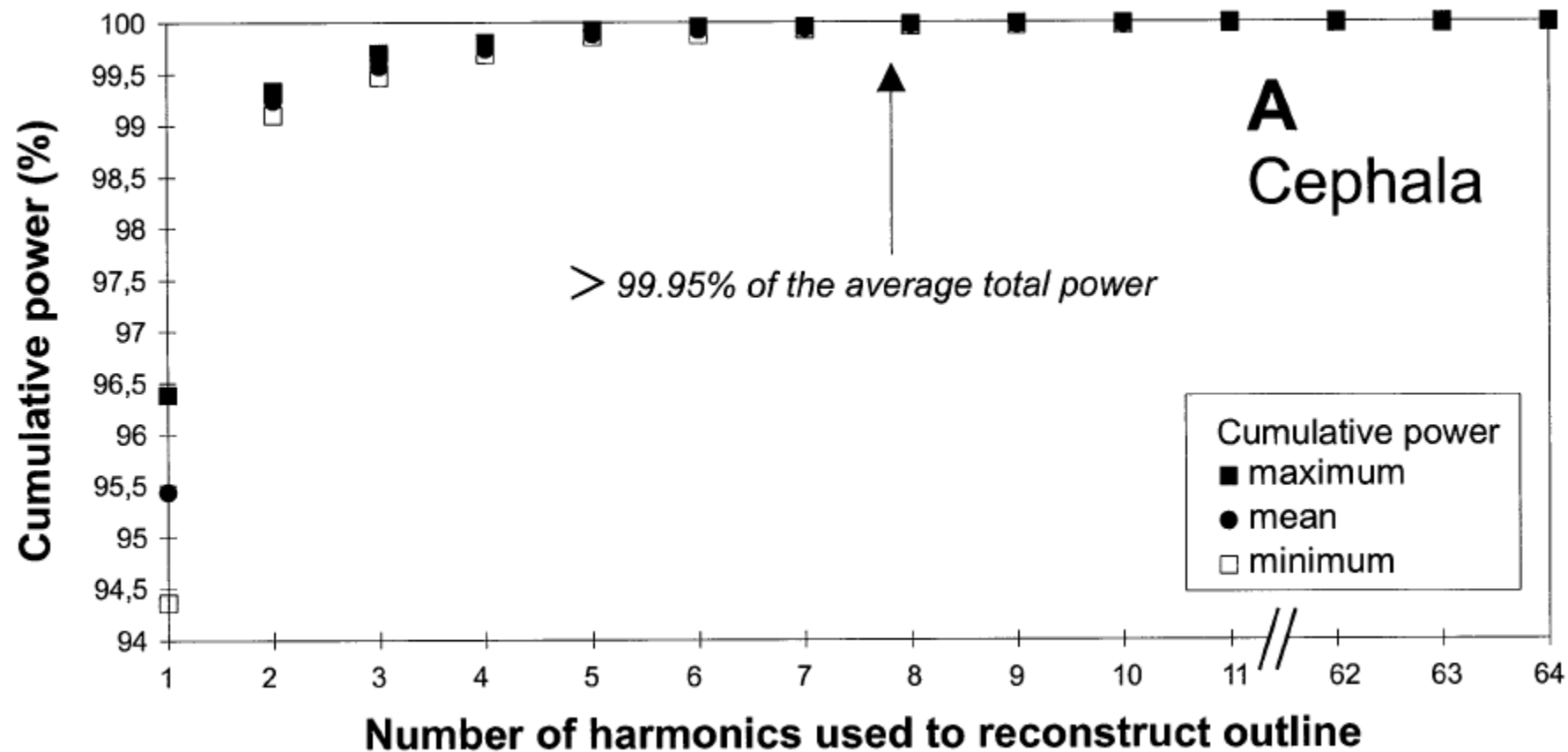


FIGURE 2. Average Fourier power (or variance) spectrum for cephalia (A) and pygidia (B). The Fourier power for the n^{th} harmonic is $(an^2 + bn^2 + cn^2 + dn^2)/2$. For any outline, the total power is calculated as the sum, from 1 to 64, of individual harmonic powers where 64 is equal to the Nyquist frequency. For both cephalia and pygidia, 99.95% of the power is described by the first eight harmonics, represented by arrows.

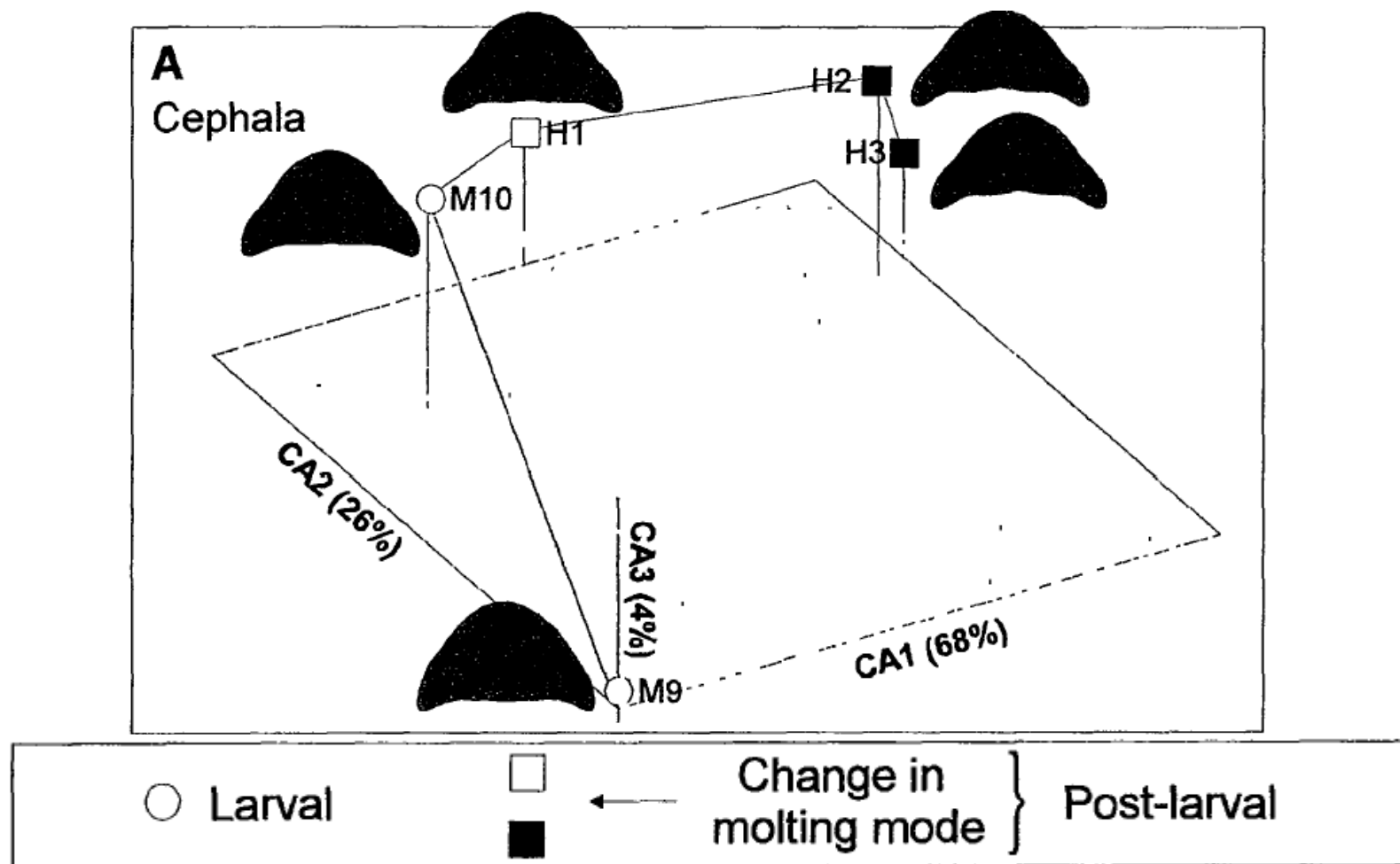
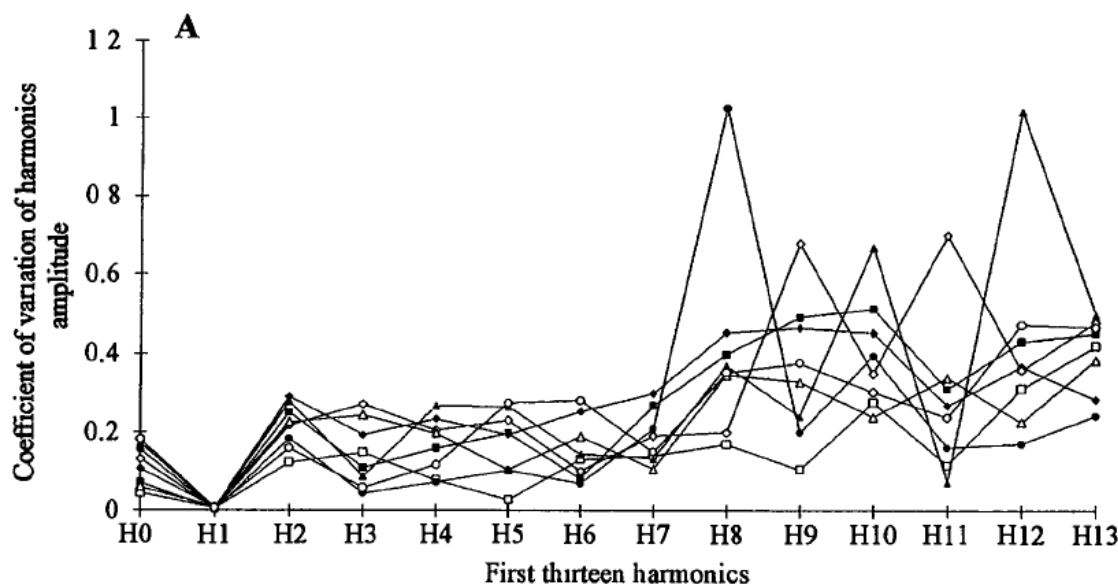


FIGURE 5. Location of the instar means and their reconstructed outlines in the canonical space corresponding to standardized Fourier coefficients. Axes CA1–3 represent 68%, 26%, and 4%, respectively, of the among-instar variance for cephalia

Fourier analysis applied to *Stephanomys* (Rodentia, Muridae) molars: nonprogressive evolutionary pattern in a gradual lineage

Sabrina Renaud, Jacques Michaux, Jean-Jacques Jaeger, and Jean-Christophe Auffray

In order to determine the rank of the last harmonic that has to be retained for a satisfactory description of the outline, we have considered the accuracy of the information provided by these harmonics. For that purpose, six repeated measurements were performed on a sample of eight upper molars. The coefficient of variation of the harmonics amplitude $a_n^2 + b_n^2 + c_n^2 + d_n^2$ increases strongly for the eighth harmonic and the subsequent ones (Fig. 3A). From this harmonic onward, the noise, caused by variation of such factors as light and positioning of the tooth during optical measurement, reduced the reproducibility of the measurement. Hence, the coefficients from the eighth harmonic onward were excluded in the statistical analysis.



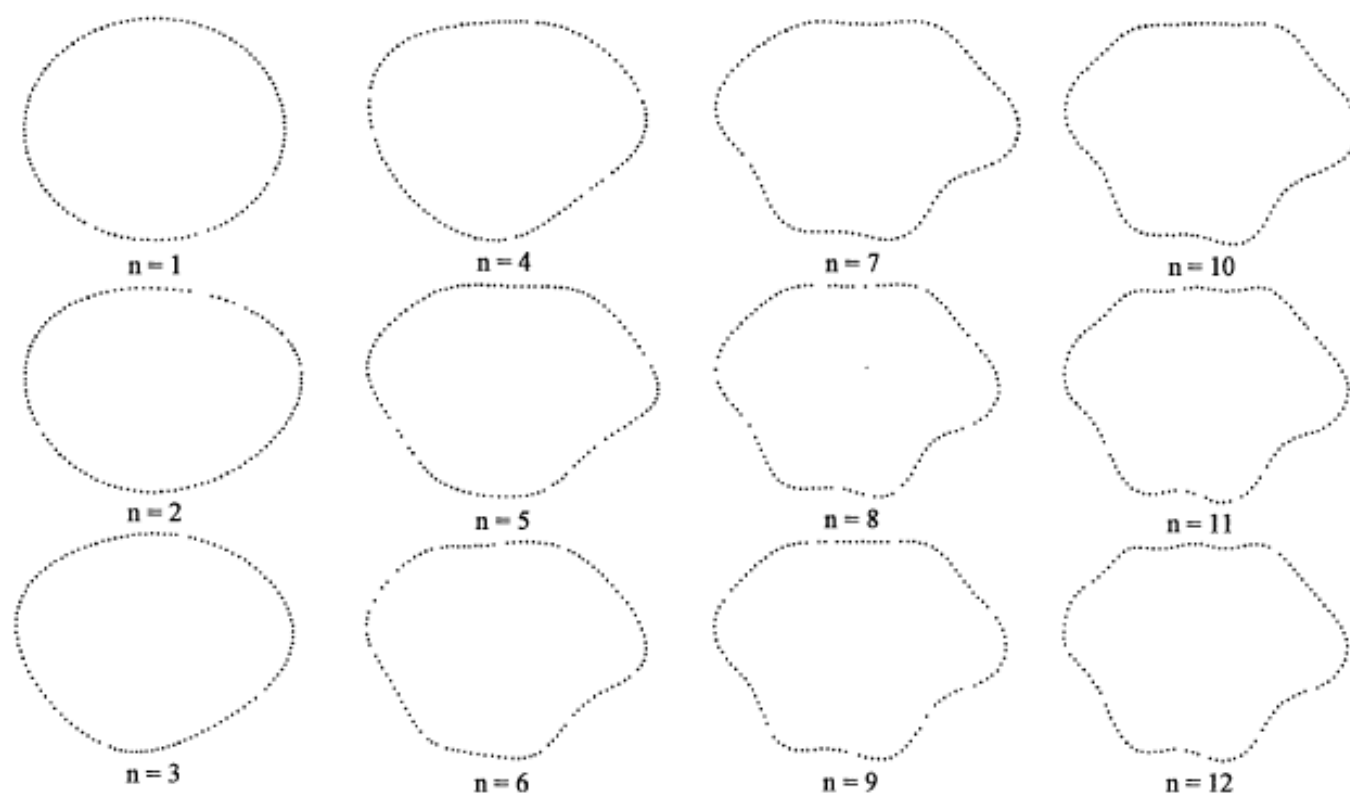


FIGURE 2. Reconstruction of a first upper molar for the successive cumulative contribution of the first twelve harmonics.

Ontogenetic and evolutionary patterns of shape differentiation during the initial diversification of Paleocene acarininids (planktonic foraminifera)

Frédéric Quillévéré, Vincent Debat, and Jean-Christophe Auffray

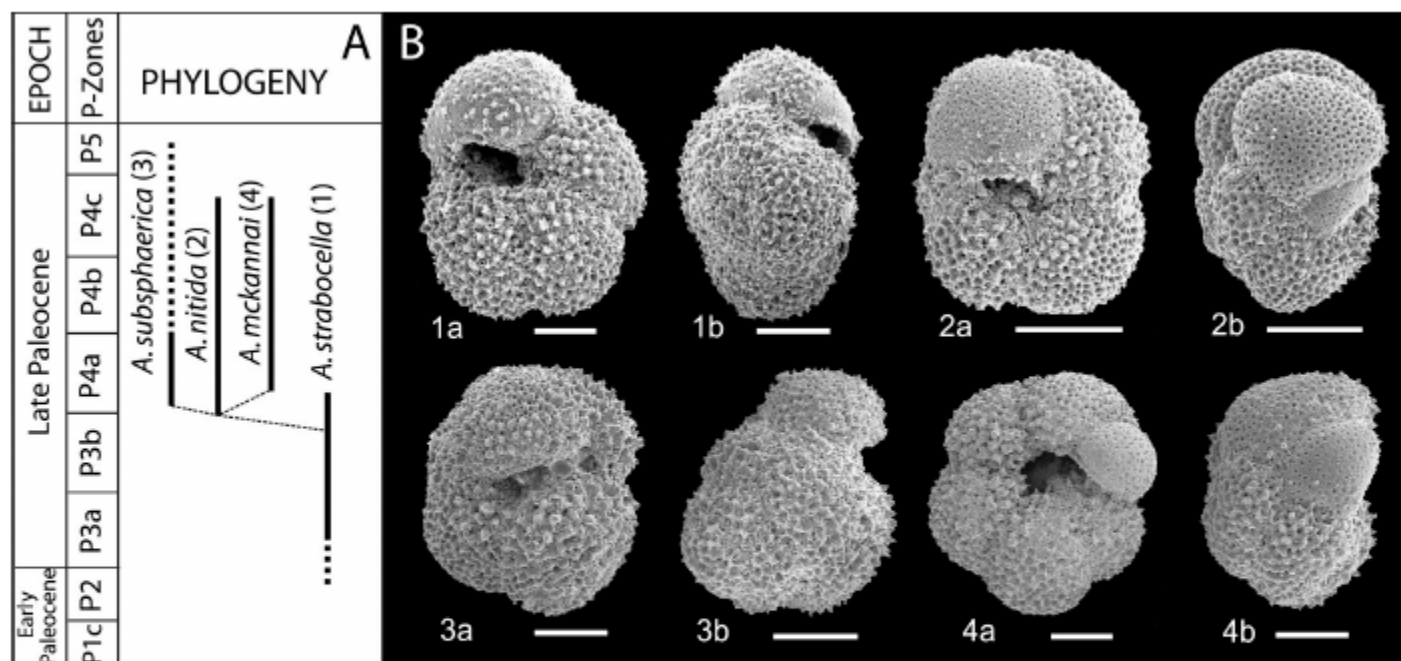


FIGURE 1. A, Phylogenetic reconstruction of Paleocene acarininids studied in this paper (dashed line represents range extension at southern high latitude sites). After Berggren and Norris (1997), Olsson et al. (1999), and Quillévéré et al. (2000). B, Scanning electron photomicrographs of representative Paleocene acarininids from ODP Hole 761B. 1: umbilical (a) and edge (b) views of *A. strabocella* from 761B-18X-2: 10–12 cm. 2: umbilical (a) and edge (b) views of *A. nitida* from 761B-18X-2: 10–12 cm. 3: umbilical (a) and edge (b) views of *A. subsphaerica* from respectively 761B-17X-4, 10–12 cm and 761B-17X-3: 10–12 cm. 4: umbilical (a) and edge (b) views of *A. mckannai* from 761B-17X-1: 10–12 cm. Scale bar, 100 μm.

SYMBOL CAPTIONS

	<i>Ac. strabocella</i>	circles: 59.0 Ma
	<i>Ac. nitida</i>	squares: 58.7 Ma
	<i>Ac. subsphaerica</i>	diamonds: 58.4 Ma
	<i>Ac. mckannai</i>	triangles: 57.2 Ma

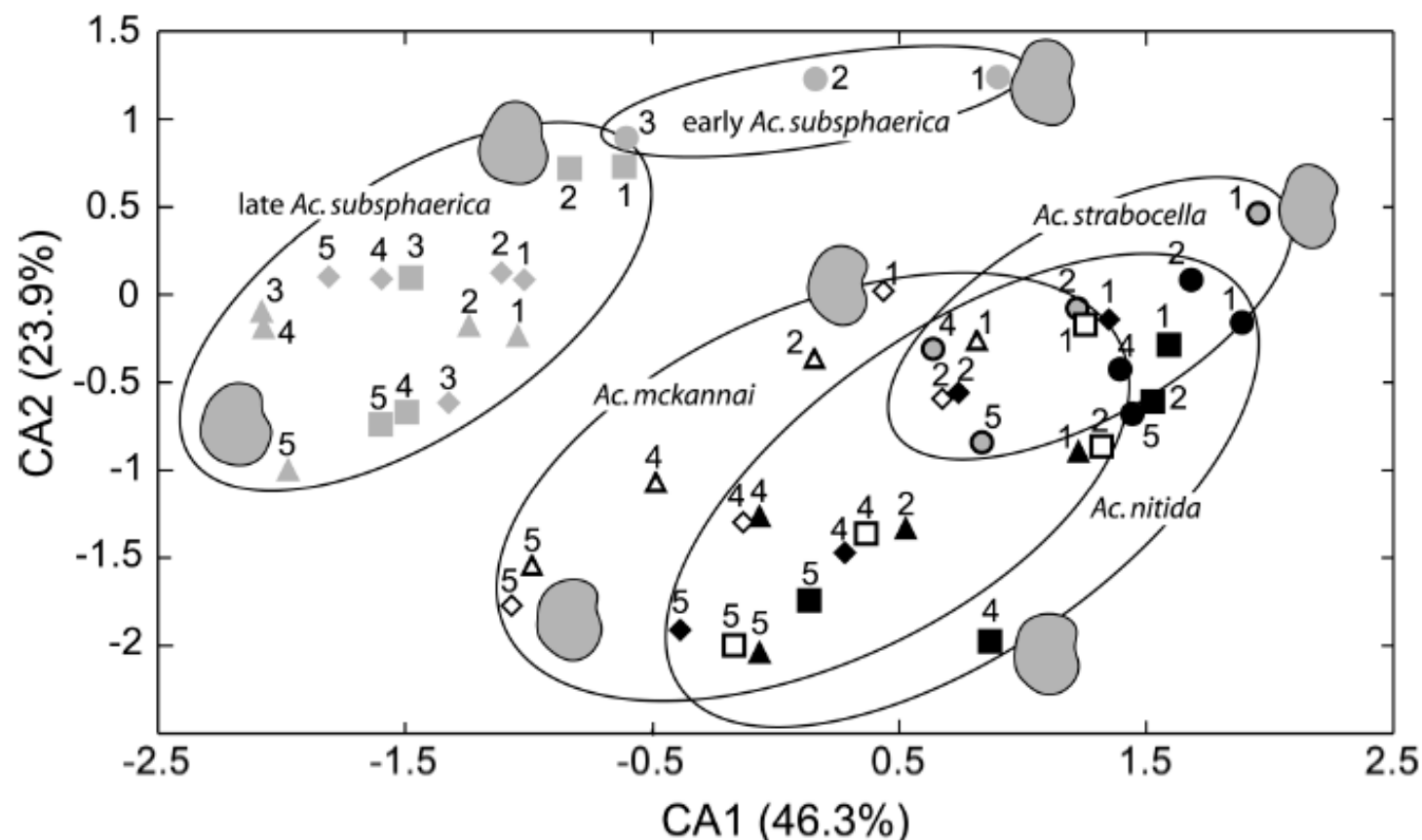


FIGURE 4. Location of the sample means in the canonical plane of the MANOVA performed on standardized Fourier coefficients of ontogenetic stages and stratigraphic horizons of the acarininid species considered in this study. The species name is expressed by the color of the symbols. The age of the samples analyzed is expressed by the shape of the symbols. Labels correspond to the different test size fractions (1 = 63–125 μm ; 2 = 125–150 μm ; 3 = 150–212 μm ; 4 = 212–250 μm ; and 5 > 250 μm). Reconstructed outlines for some group means visualize shape changes in the plane defined by the first two canonical axes. These reconstructed mean outlines of selected groups have been obtained using inverse Fourier Transform.

Evaluation of variation of root shape of Japanese radish (*Raphanus sativus* L.) based on image analysis using elliptic Fourier descriptors

Hiroyoshi Iwata¹, Satoshi Niikura², Seiji Matsuura², Yasushi Takano¹ & Yasuo Ukai¹

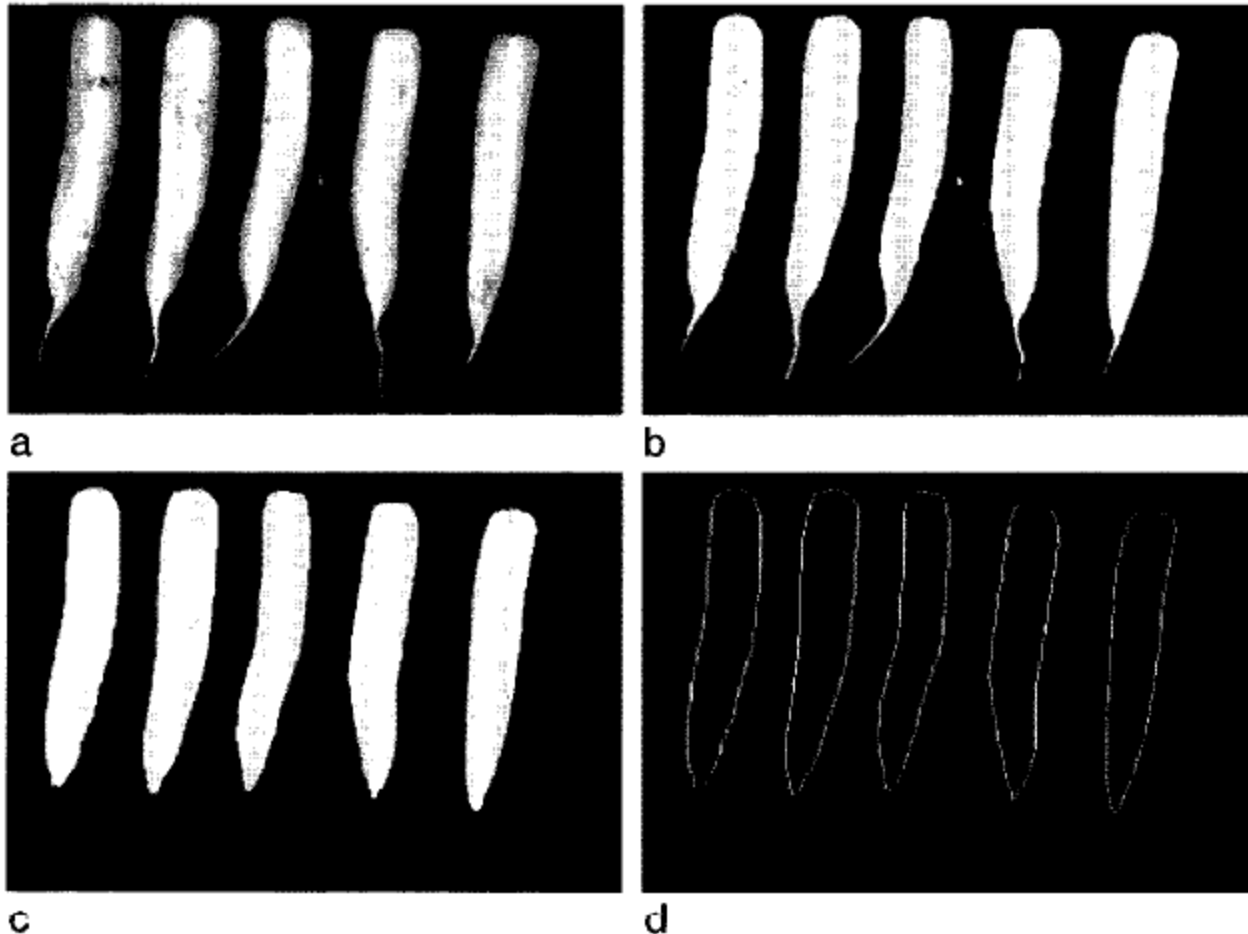


Figure 2. Series of image processing which extracts chain-coded contour from red digital image; a: red digital image of 256 gray levels; b: binary image; c: image after noise reduction, erosion, dilation, and filling holes; d: extracted contours

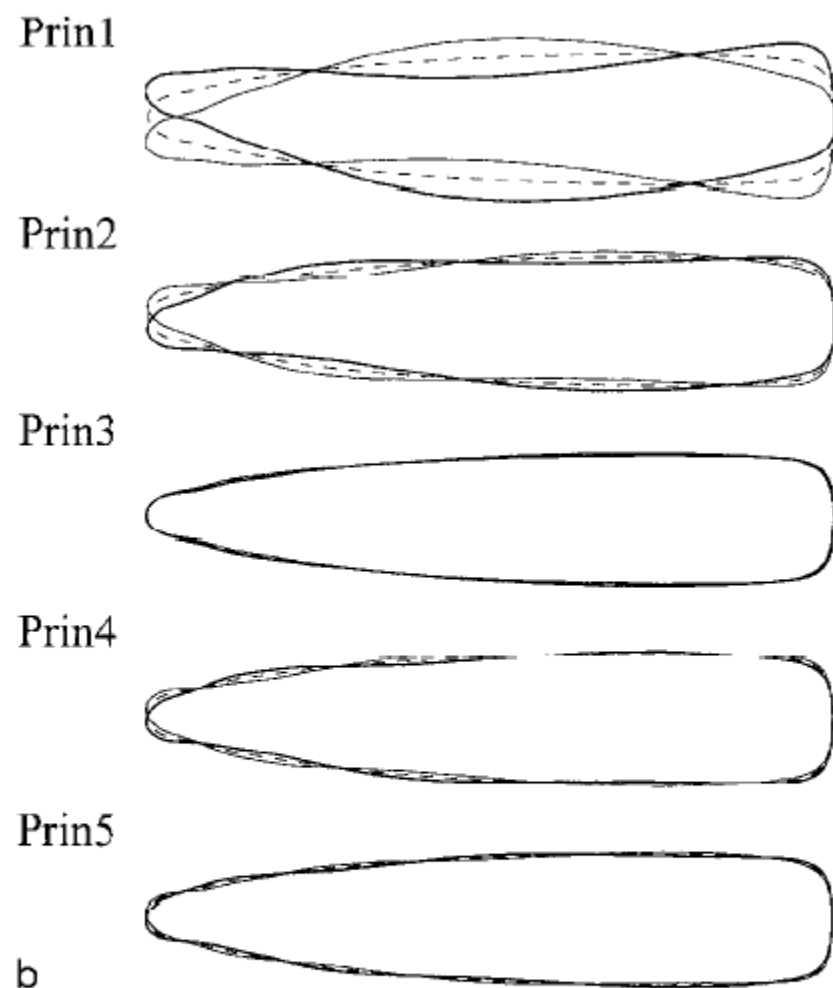
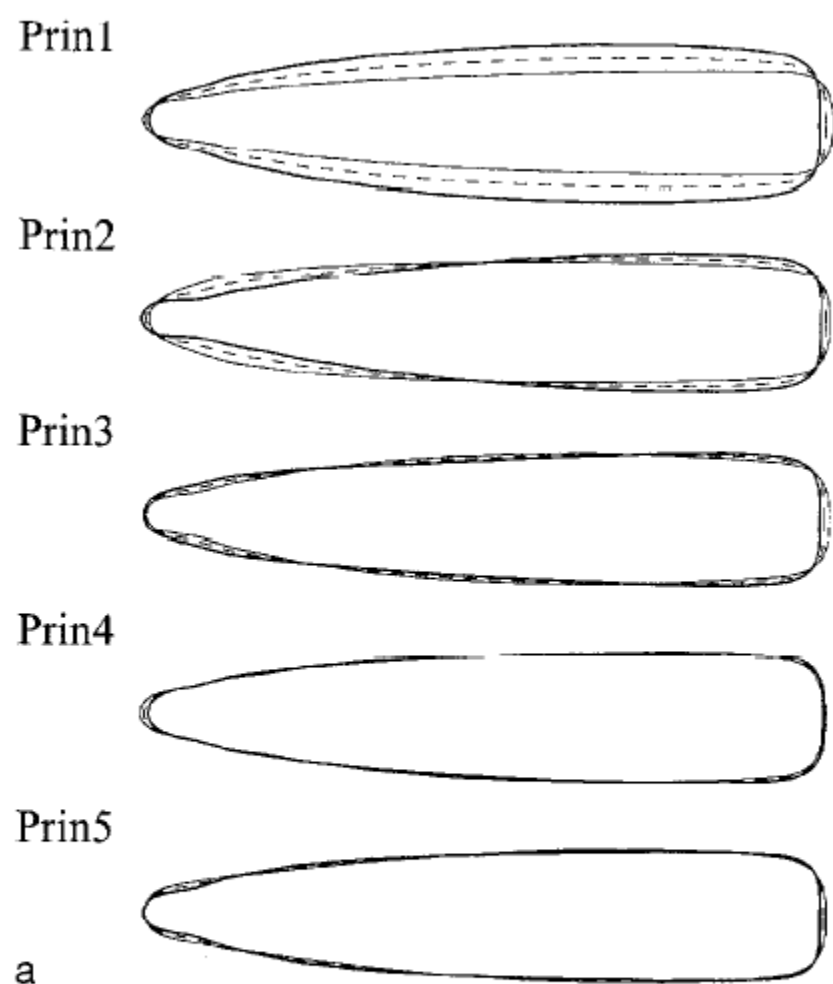


Figure 3. The shape variation which can be accounted for by each principal component. Each shape was reconstructed from the coefficients which were calculated by letting the score on the corresponding principal component be equal to mean with plus or minus two times standard deviation and the scores on the remaining component zero. (a) symmetrical variation from group A coefficients. (b) asymmetrical variation from group B coefficients. Dashed line, thick solid line, and thin solid line stand for mean, mean + 2S.D., mean - 2S.D.



Geographic variation and plasticity of leaf shape and size in *Begonia dregei* and *B. homonyma* (Begoniaceae)

TRACY McLELLAN FLS*

Department of Molecular and Cell Biology, University of the Witwatersrand, Private Bag 3, Wits 2050, South Africa

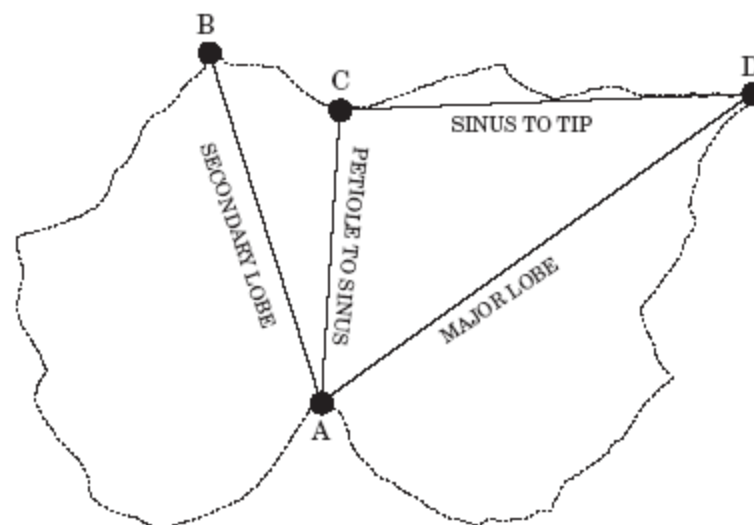


Figure 2. Measurement taken on digitized leaf outlines. Point A was the first point in the outline at the junction of the petiole and the lamina; points B and C were designated during digitizing, and point D was found by finding the point on the outline at the greatest distance from point A. Part perimeter is the distance along the perimeter between points C and D.

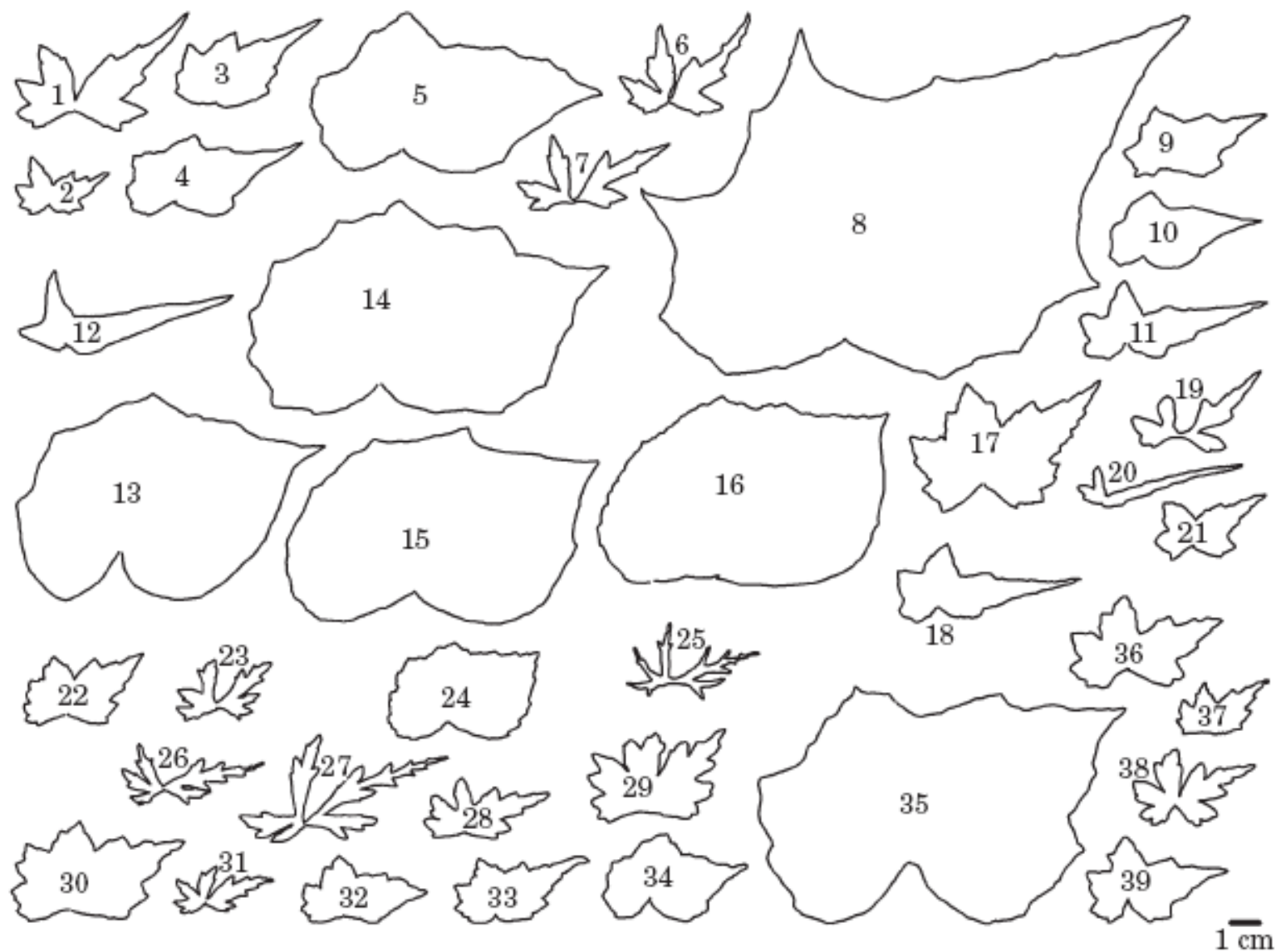


Figure 3. Silhouettes of representative leaves from each of 39 populations. Population numbers are as in Table 1.

The first six harmonics of elliptic Fourier coefficients (EFC) were calculated, normalized, and the power series was calculated from the x, y coordinates of the images (McLellan & Endler, 1998). These coefficients were then averaged over all the leaves taken from each plant, and the means of the coefficients were used in the principal component analysis.

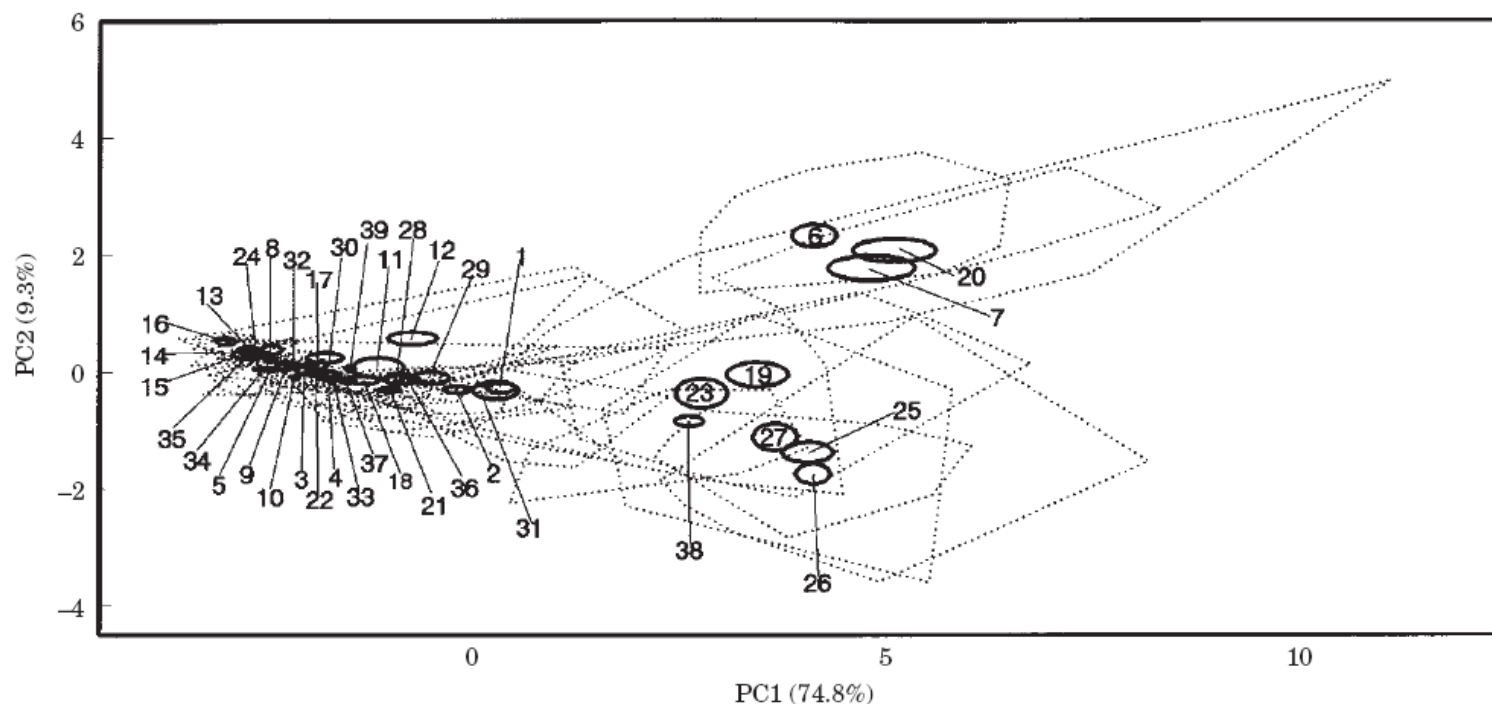


Figure 6. Plot of the first two principal components of the first five harmonics of elliptic Fourier coefficients for 39 population samples.

Ellipses are one standard error from the means of the PCs for each population, and polygons are defined by samples with extreme values for each population.