

NMR spectroscopy

- vector model – energy levels model
- chemical shift
- J interaction
- equivalence
- interpretation

Nuclear spin and magnetic moment

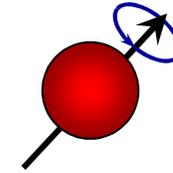
$$\vec{I} = (I_x, I_y, I_z)$$

Nuclear angular momentum

intrinsic

$$|\vec{I}| = \hbar\sqrt{I(I+1)}$$

spin number



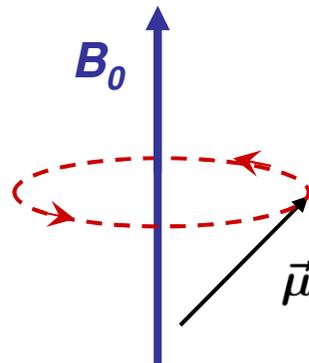
Magnetic moment

$$\vec{\mu} = \gamma\vec{I}$$

gyromagnetic ratio

Magnetic moment in magnetic field

$$\vec{B} = (0, 0, B_0)$$



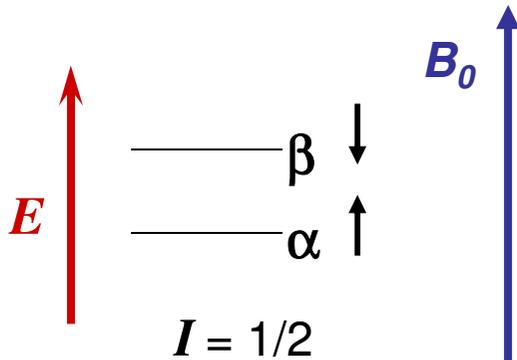
Larmor precession

$$\nu = -\frac{\gamma}{2\pi}B_0$$

Ensemble of spins

Energy of magnetic moment in magnetic field

$$E = -\vec{\mu} \cdot \vec{B}$$



Boltzman distribution of energies

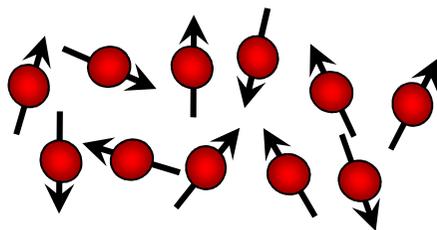
$$\Delta E = \hbar \gamma B_0$$

$$\frac{N_\alpha}{N_\beta} = \exp \left\{ \frac{\Delta E}{k_b T} \right\}$$

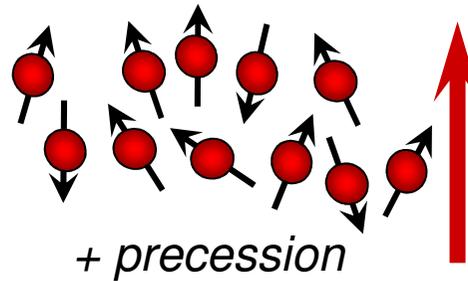
^1H , temperature 300K, magneticfield induction 9.4 T

$$\frac{N_\alpha}{N_\beta} = 1.000064$$

$B_0 = 0$



B_0

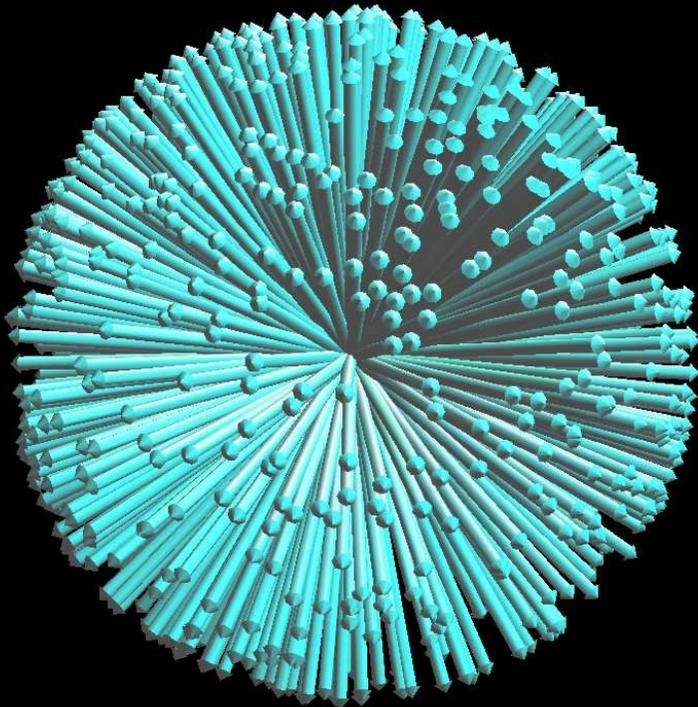


Magnetization
sum of magnetic moments

Slight preference of parallel orientation

Ensemble of spins

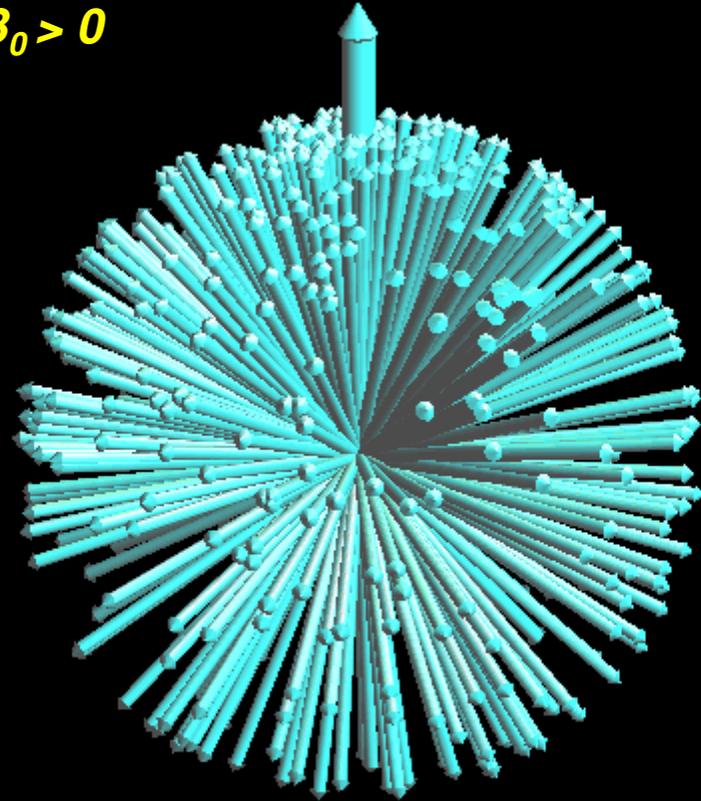
$B_0 = 0$



$$\vec{M} = 0$$

random orientations

$B_0 > 0$



$$\vec{M} = (0, 0, M_0)$$

precession and polarization

random initial phase

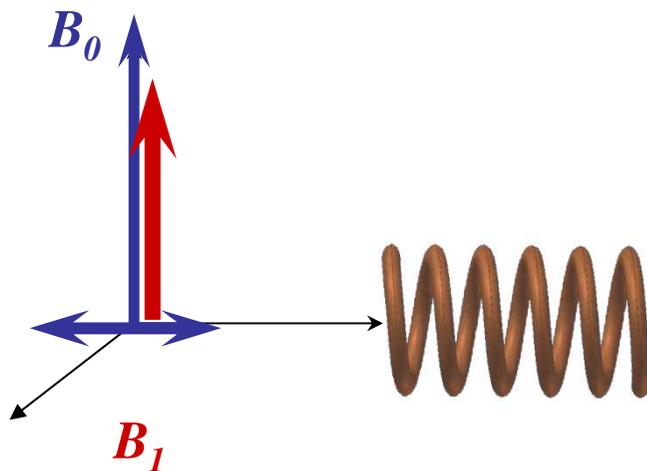
Bloch equations

Behavior of magnetization in magnetic field (general)

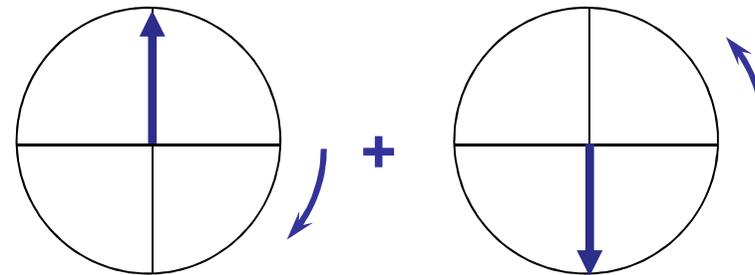
$$\frac{d}{dt}M_z(t) = \underbrace{\gamma \left[\vec{M}(t) \times \vec{B}(t) \right]_z}_{\text{precession}} - \underbrace{\frac{M_z(t) - M_0}{T_1}}_{\text{relaxation}} \quad \text{longitudinal}$$

$$\frac{d}{dt}M_{x,y}(t) = \gamma \left[\vec{M}(t) \times \vec{B}(t) \right]_{x,y} - \frac{M_{x,y}(t)}{T_2} \quad \text{transversal}$$

Static and RF fields



**Rotating frame
in resonance with Larmor precession**

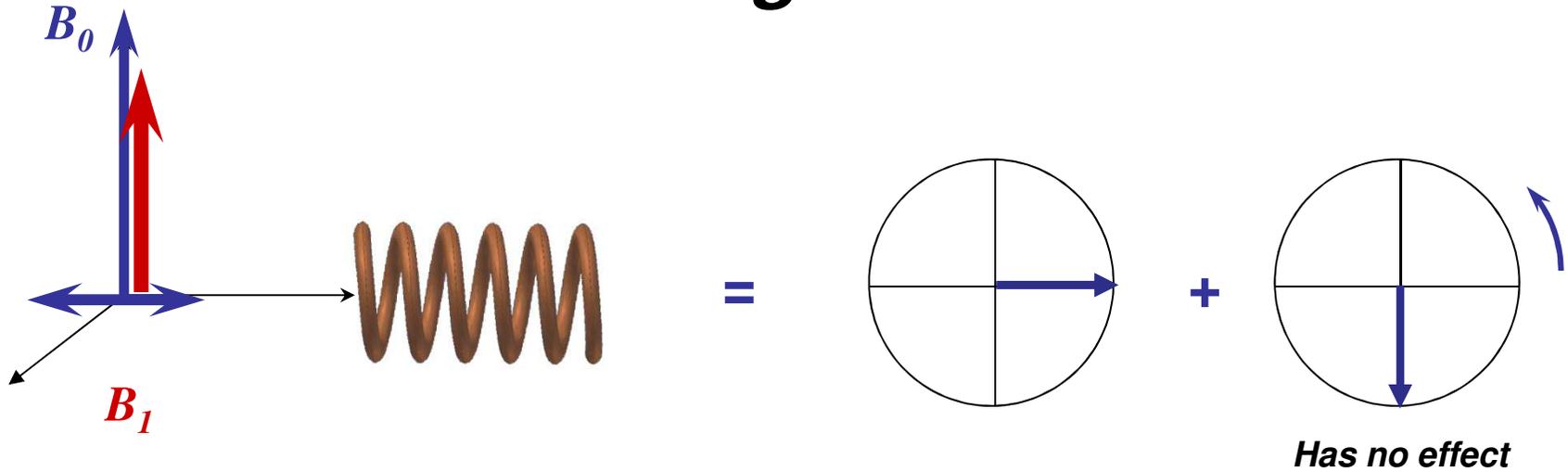


$$(B_1 \cos \omega_{RF}t, -B_1 \sin \omega_{RF}t, 0)$$

$$(B_1 \cos \omega_{RF}t, B_1 \sin \omega_{RF}t, 0)$$

$$(2B_1 \cos \omega_{RF}t, 0, 0)$$

Rotating frame



Total magnetic field in rotating frame

$$B = \left(B_1, 0, B_0 - \frac{\omega_{RF}}{\gamma} \right)$$

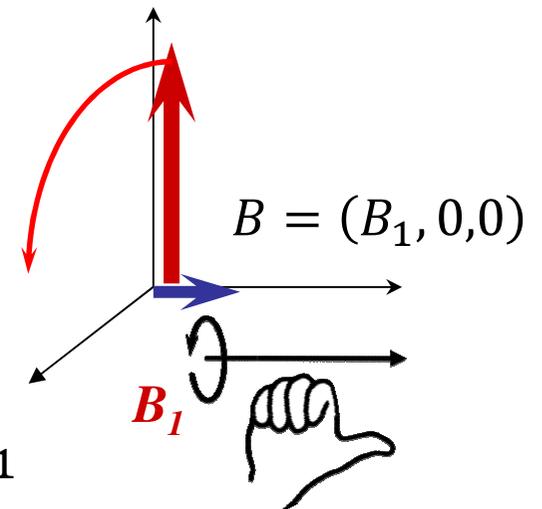
Resonance condition

$$\omega_{RF} = \gamma B_0$$

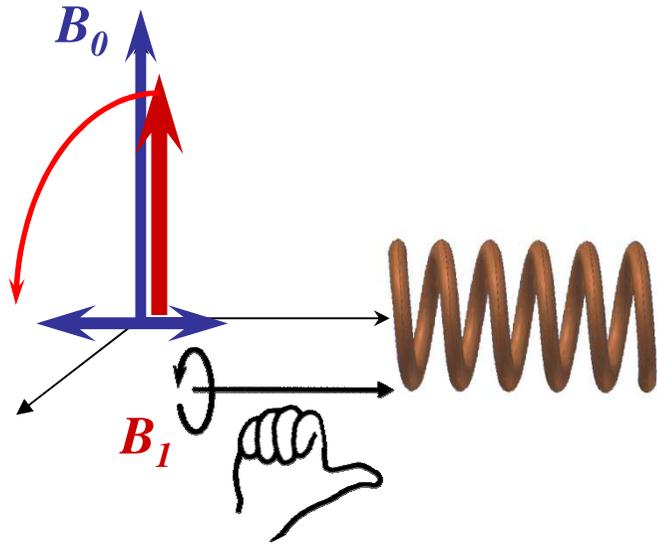
Magnetization feels only static B_1

nutations

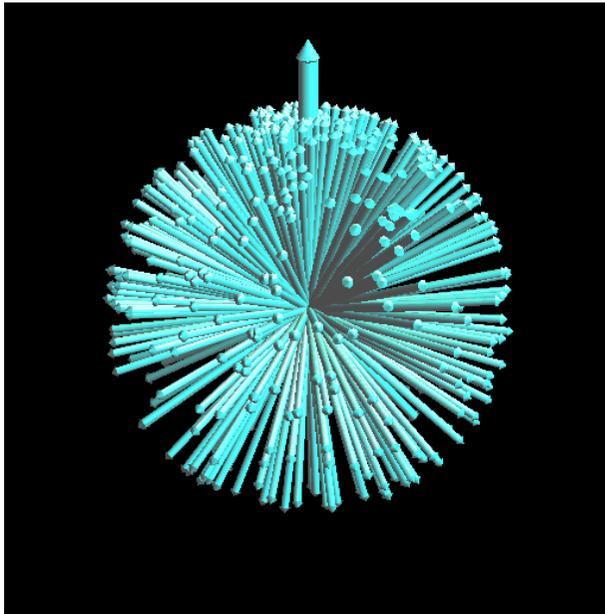
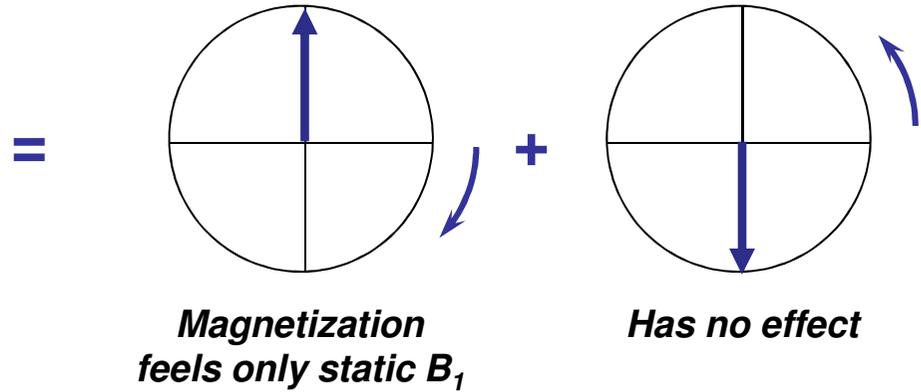
$$\omega_1 = \gamma B_1$$



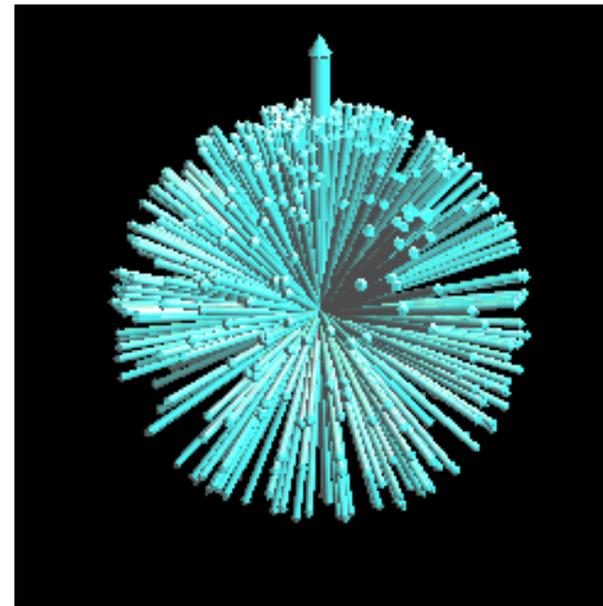
Effect of radiofrequency field



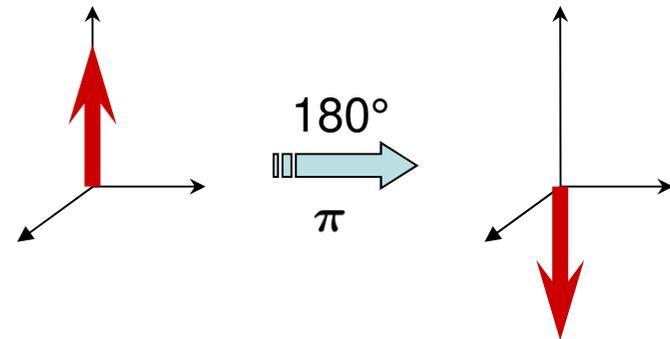
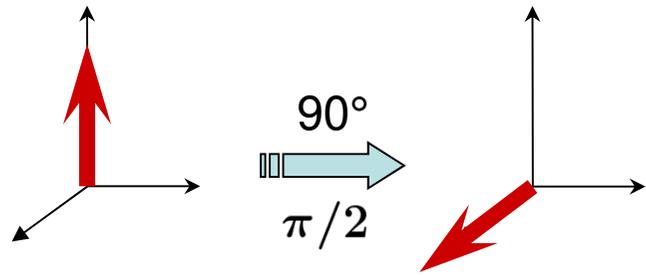
*Rotating frame
in resonance with Larmor precession*



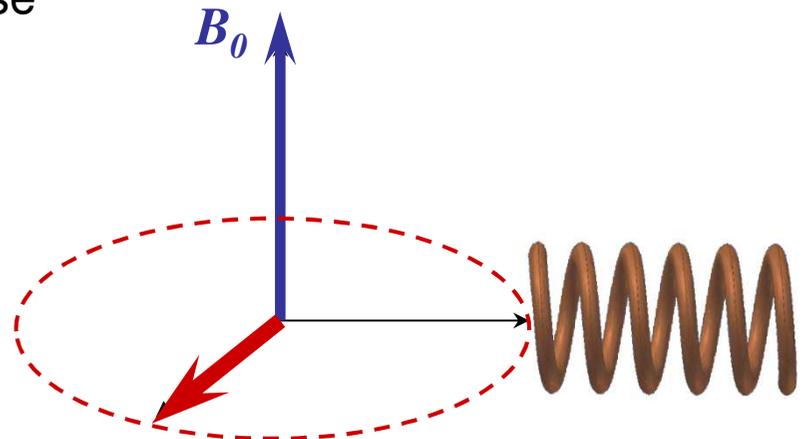
$$\omega_{RF} = \gamma B_0$$



RF pulses and NMR signal



After 90° pulse

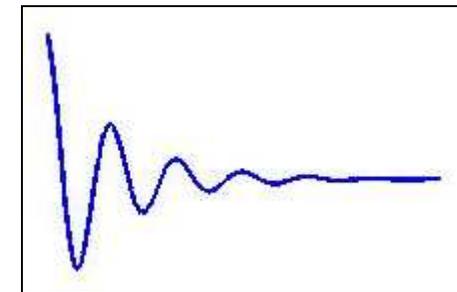


Larmor precession

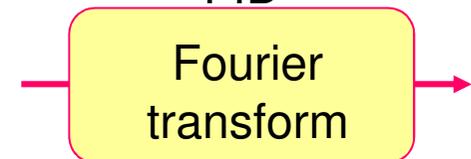
$$\nu = -\frac{\gamma}{2\pi} B_0$$



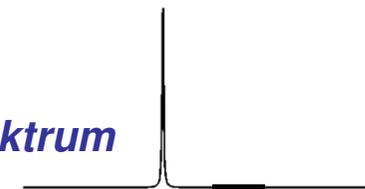
time-variable magnetic flux



Free Induction Decay
FID

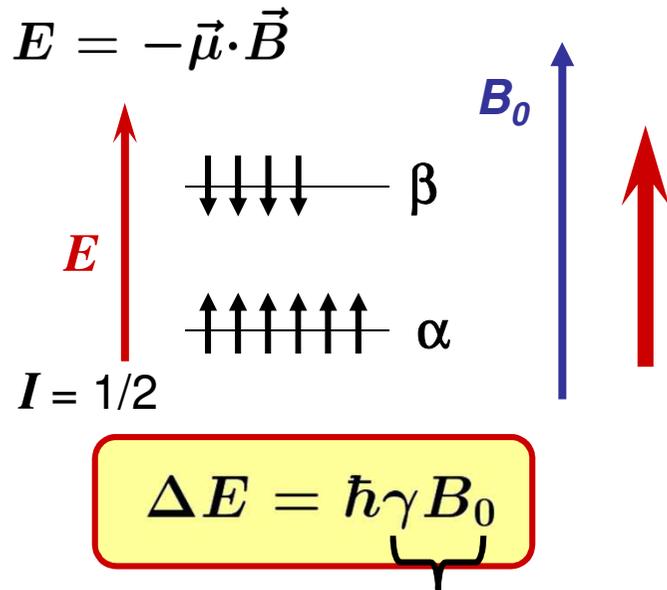


NMR spektrum



Quantum physics view on NMR

Full description, might not always be intuitive



Larmor frequency

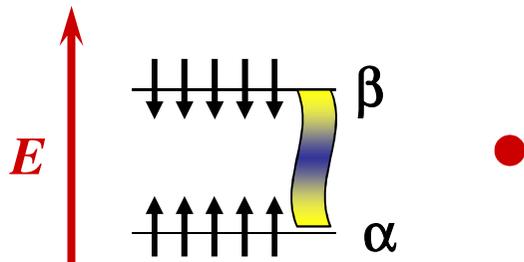
Boltzman distribution of energies

$$\frac{N_\alpha}{N_\beta} = \exp \left\{ \frac{\Delta E}{k_b T} \right\}$$

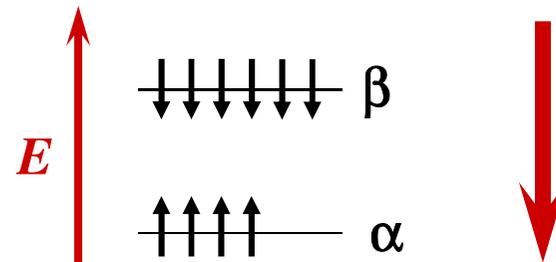
$$\|\vec{M}\| \propto N_\alpha - N_\beta$$

NMR signal proportional to initial magnetization
– difference of populations on energy levels

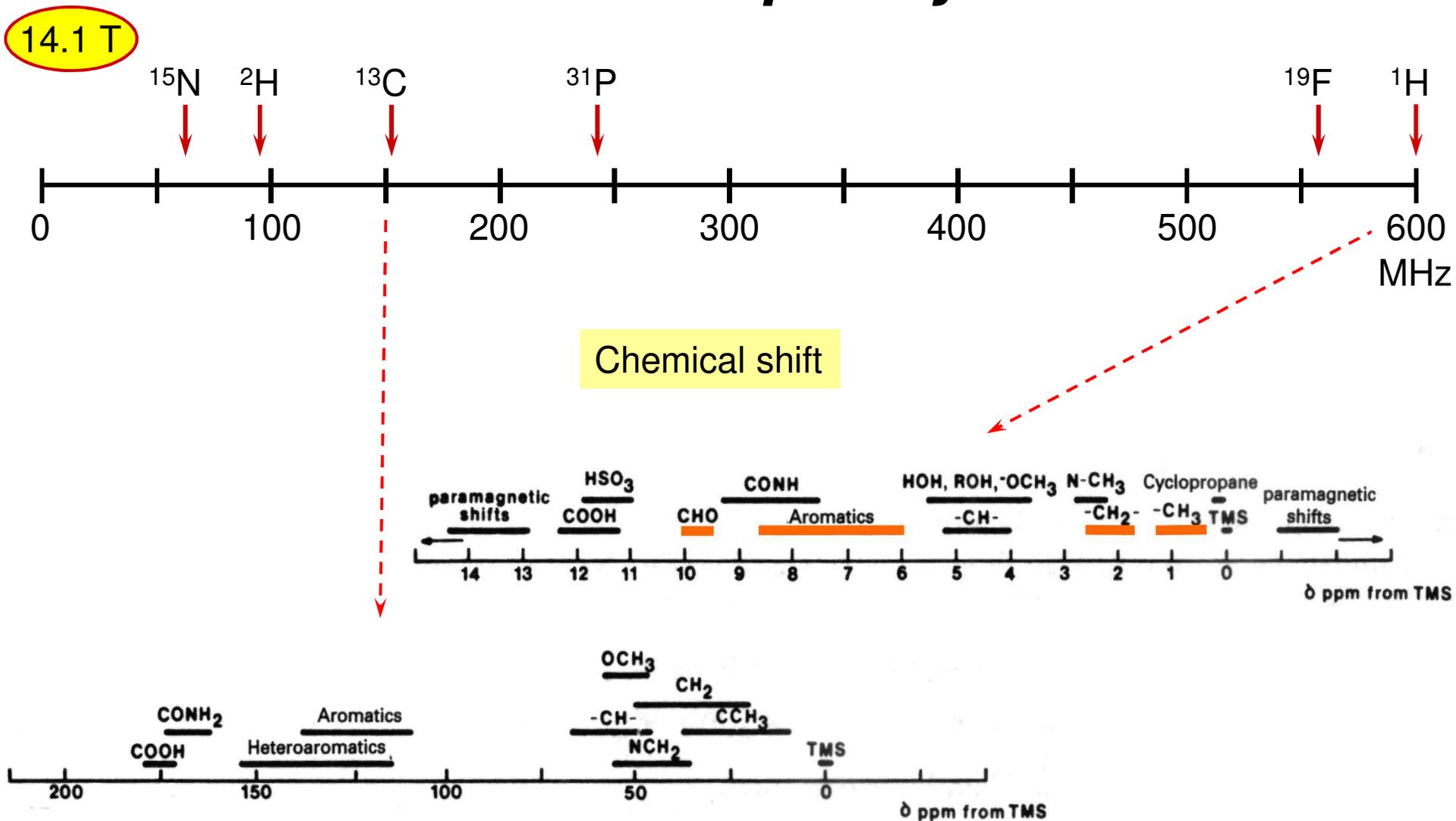
90°-pulse creates a **coherence**
equalizes populations



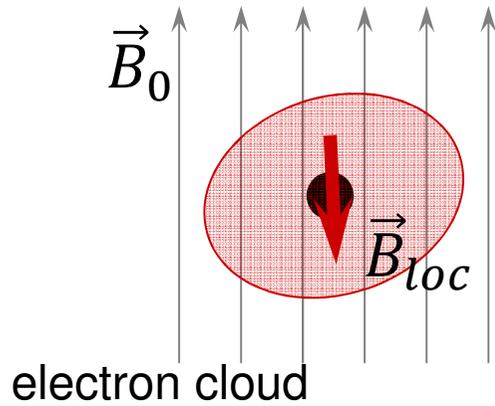
180°-pulse
inversion of populations



NMR frequency

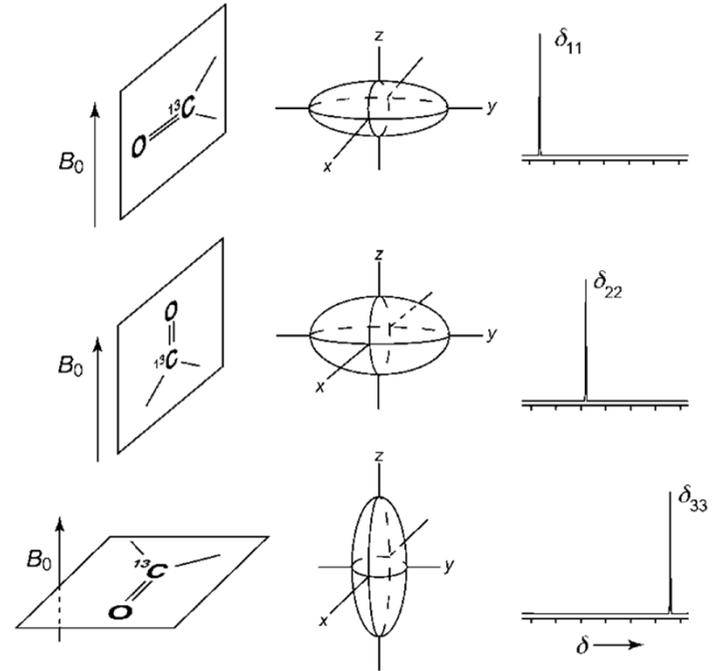
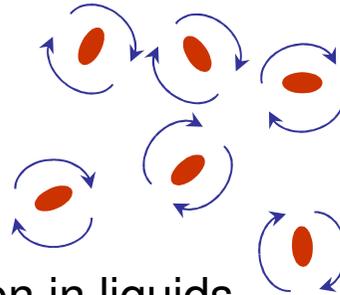


Chemical shielding



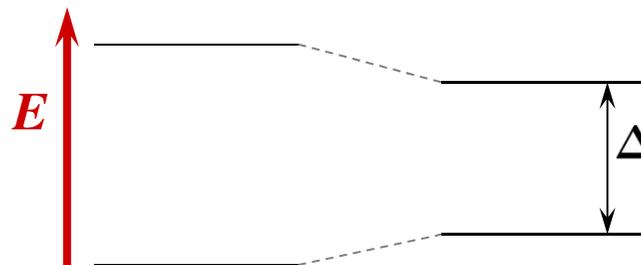
$$\vec{B}_{loc} = -\vec{\sigma} \vec{B}_0$$

shielding tensor



$$B = B_0 + B_{loc} = B_0(1 - \sigma_{iso})$$

shielding constant



resonance frequency

Chemical shift

$$\delta = 10^6 \frac{\nu - \nu_{ref}}{\nu_{ref}}$$

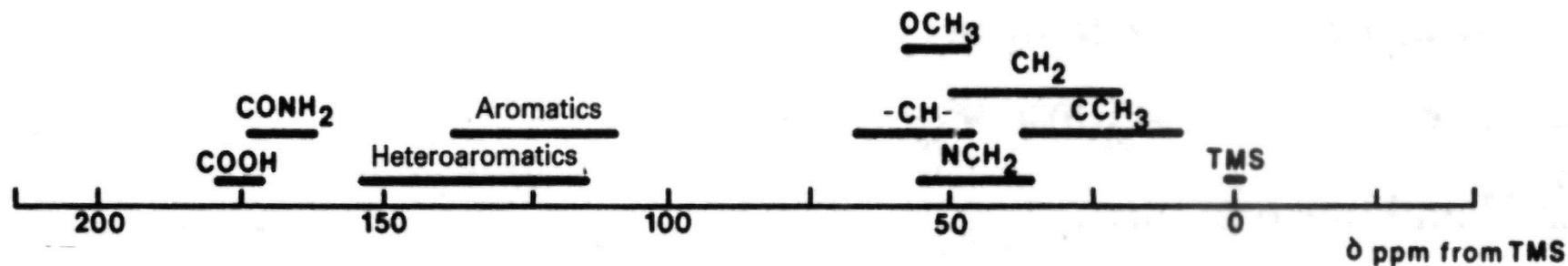
ppm

parts per milion

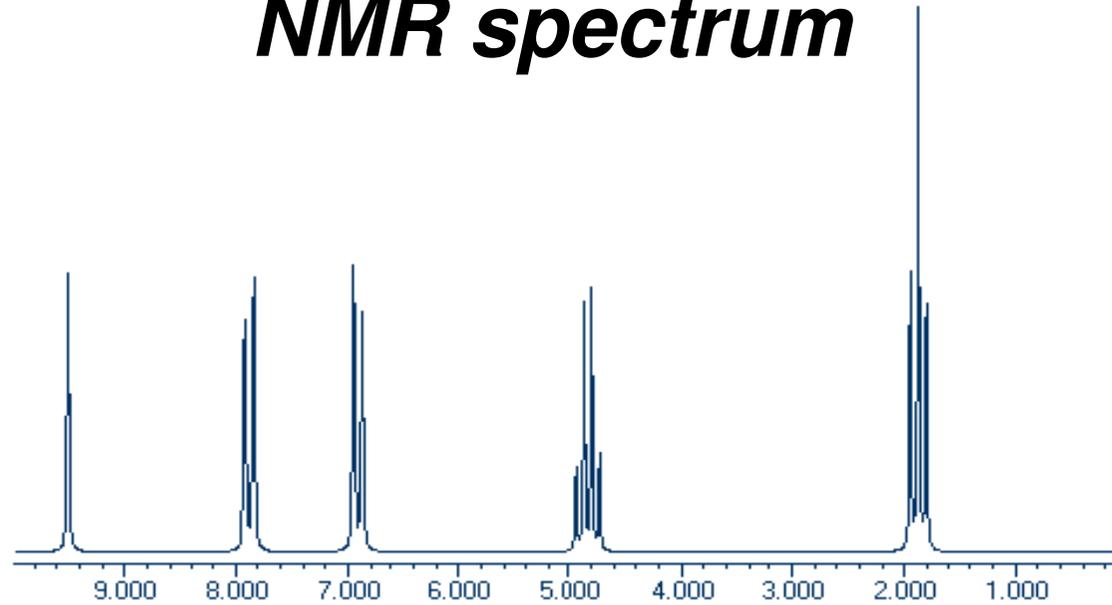
¹H



¹³C



NMR spectrum



← ppm, frequency →

→ chemical shielding ←

high δ
low field
downfield
paramagnetic shift
deshielding

low δ
high field
upfield
diamagnetic shift
shielding

Chemical shielding

$$\sigma = \sigma^{dia} + \sigma^{para} + \sigma^{local}$$

dia electrons in s-orbitals, decreasing local magnetic field

para π -electrons and in p-orbitals, increasing local magnetic field

local influence of surrounding substituents, both positive and negative
strongest

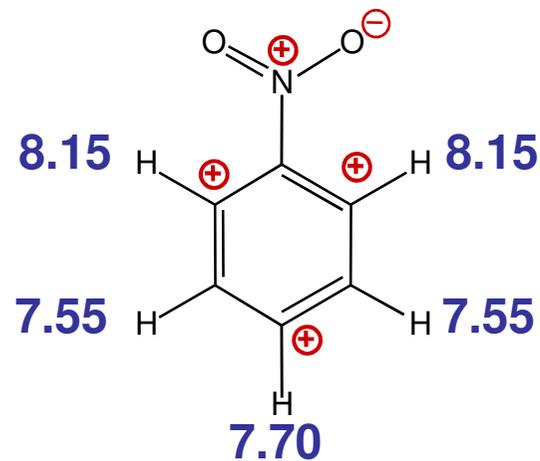
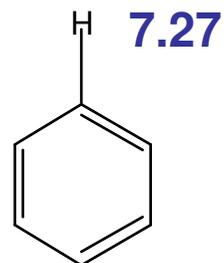
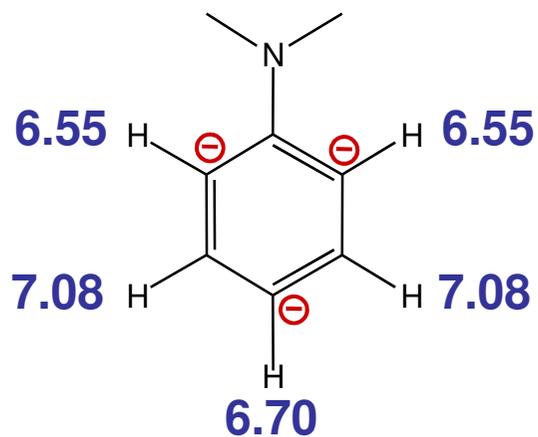
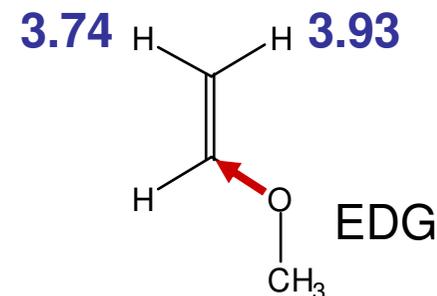
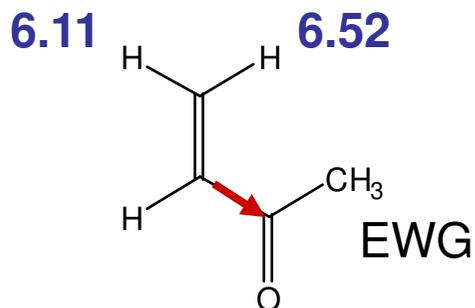
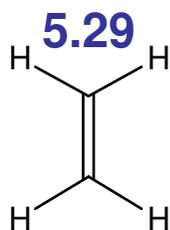
General principle

The more electrons around a nucleus the higher shielding and lower chemical shift

Proton chemical shifts

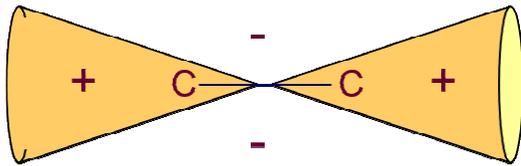
	CH ₄	CH ₃ I	CH ₃ Br	CH ₃ Cl	CH ₃ F
Electronegativity	2.1	2.5	2.8	3.0	4.0
Shift [ppm]	0.23	1.98	2.45	2.84	4.13

Resonance (mezomeric) effect

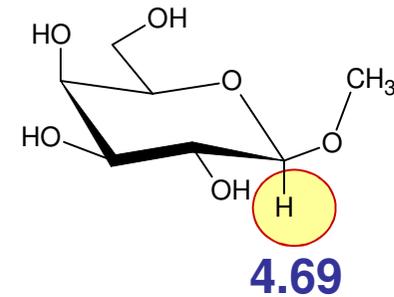
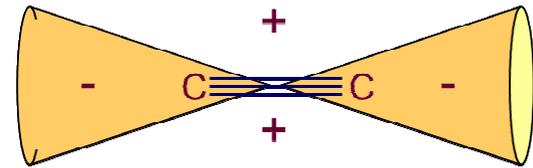
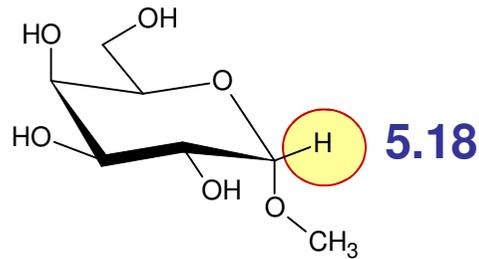
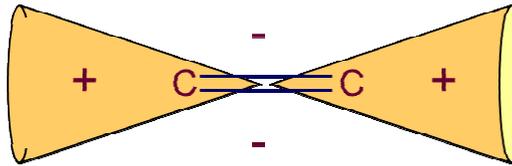


Proton chemical shifts

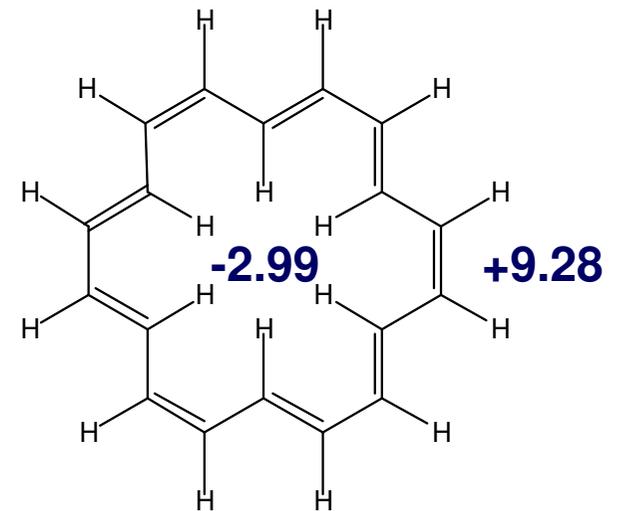
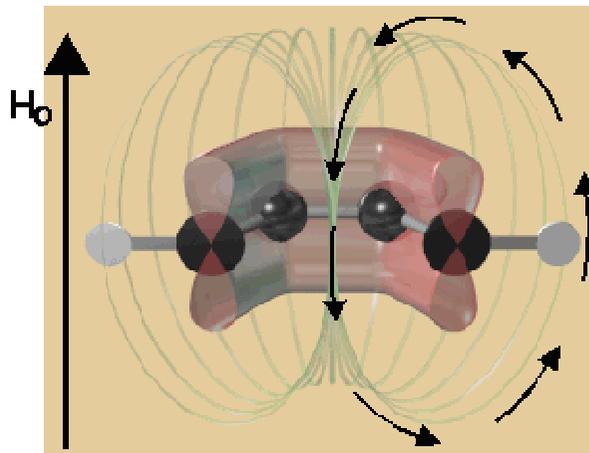
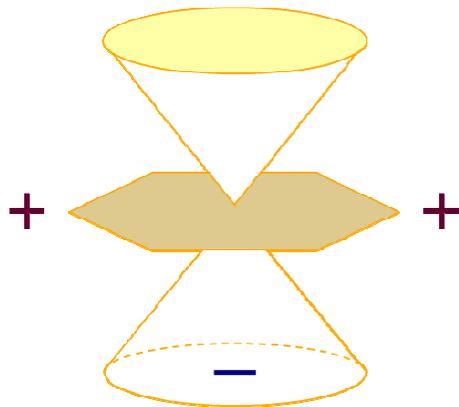
Anisotropic effect



+ higher δ
- lower δ



Ring current effect



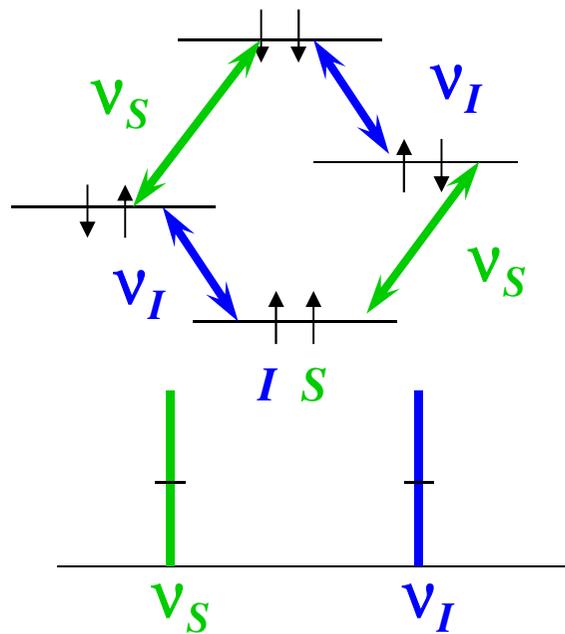
J interaction

Indirect spin-spin coupling

through common electrons

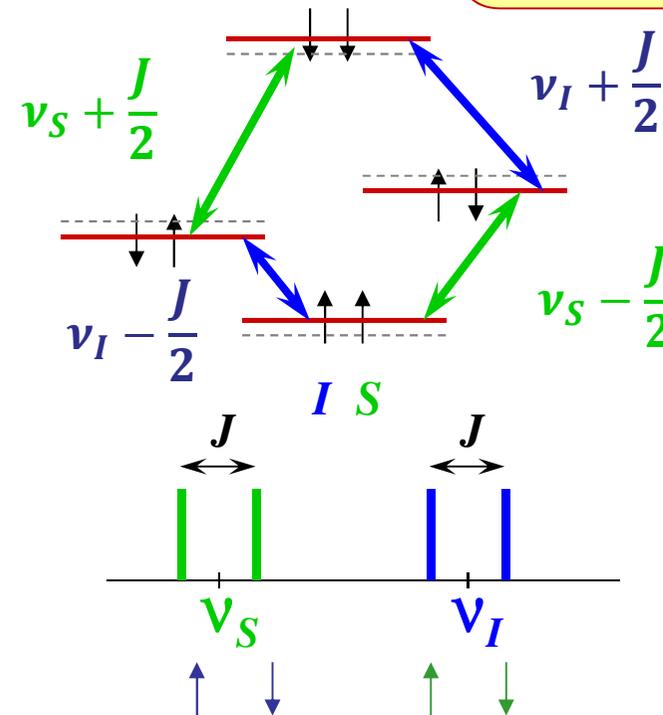


Two spins-1/2, no interaction



both transitions equal

including J coupling

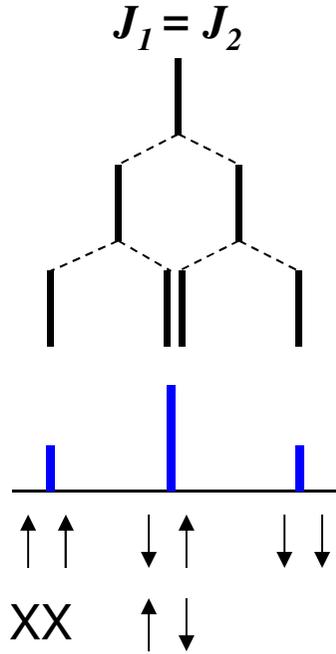


**energy
contribution**

$$E_J = h J m_I m_S$$

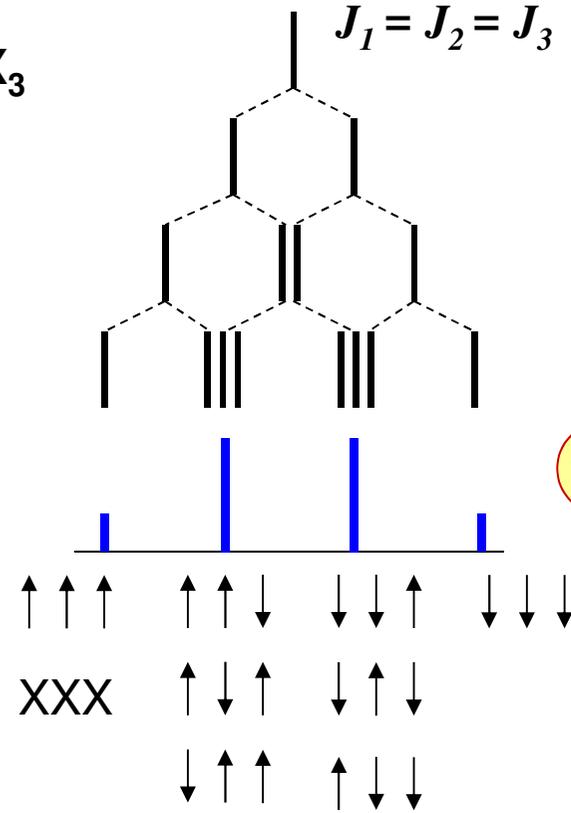
J interaction

AX₂



t

AX₃



q

All for spins-1/2

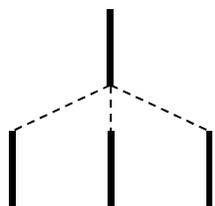
<i>n</i>		1				singlet s	
1		1	1			doublet d	
2		1	2	1		triplet t	
3		1	3	3	1	quartet q	
4		1	4	6	4	1	pentet p (quintet)

Pascal triangle

+ combinations

J interaction

AX



1 : 1 : 1

CDCl_3

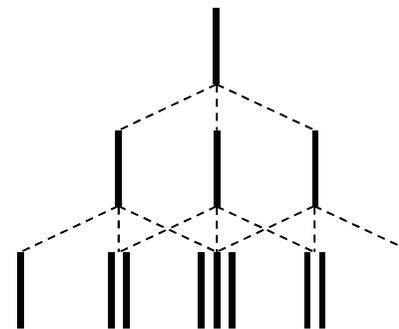
... also for NH_4^+

When X is spin-1

possible states

-1, 0, +1

AX₂



1 : 2 : 3 : 2 : 1

CD_2Cl_2

11B (80%) : spin-3/2

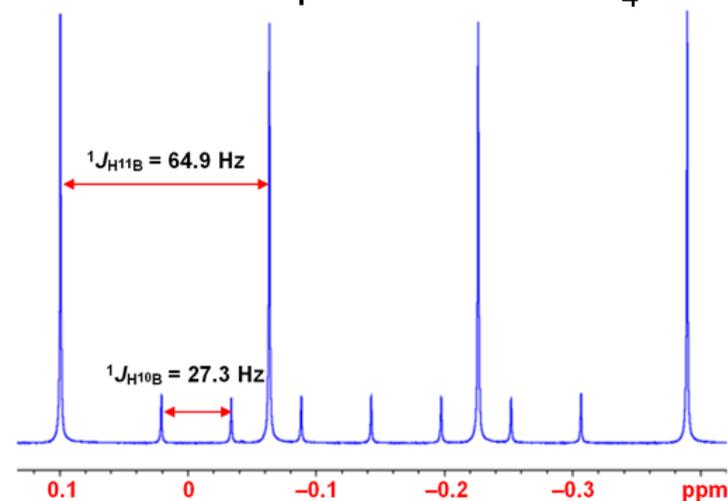
possible states

-3/2, -1/2, +1/2, +3/2

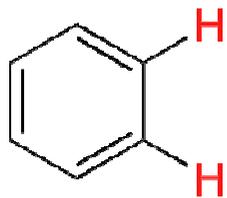
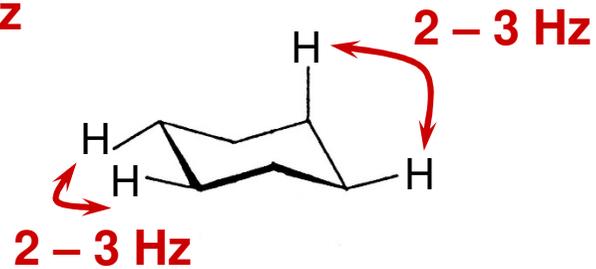
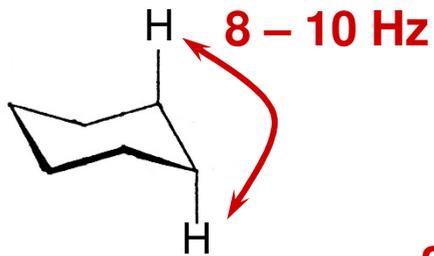
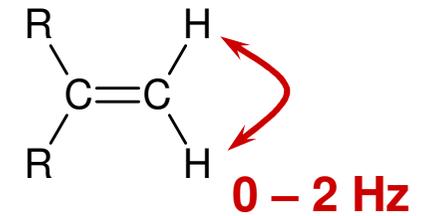
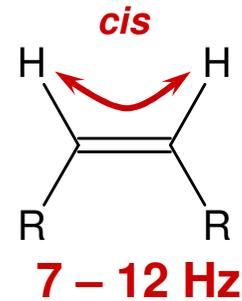
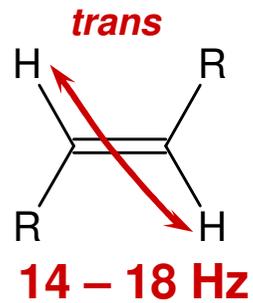
10B (20%) : spin-3

possible states

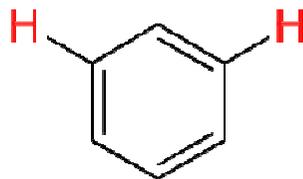
Proton spectrum of $^+\text{BH}_4$



J interaction (proton-proton)

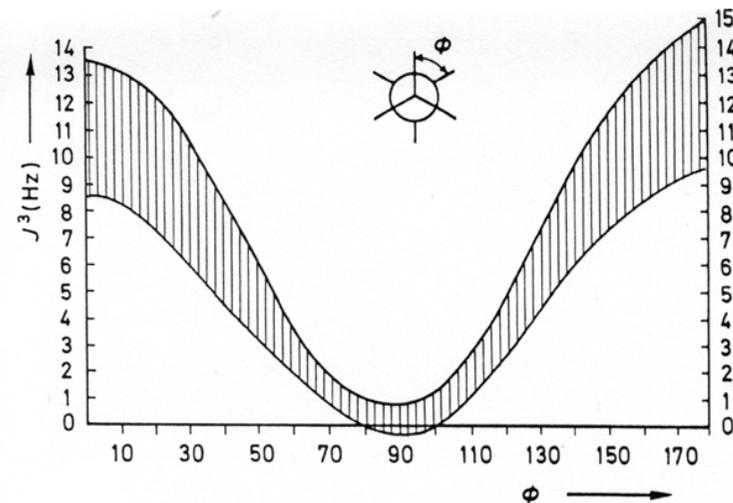


ortho-benzylic:
 $^3J = 6-10$ Hz



meta-benzylic:
 $^4J = 0-4$ Hz

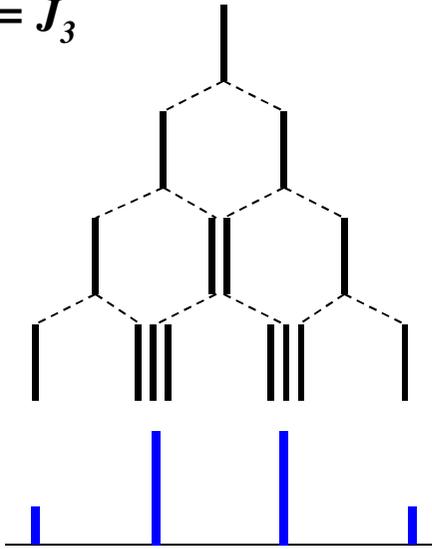
Karplus curve



$$^3J = \begin{cases} 8,5 \cos^2 \phi - 0,28 & \text{for } 0 \leq \phi \leq 90^\circ \\ 9,5 \cos^2 \phi - 0,28 & \text{for } 90^\circ \leq \phi \leq 180^\circ \end{cases}$$

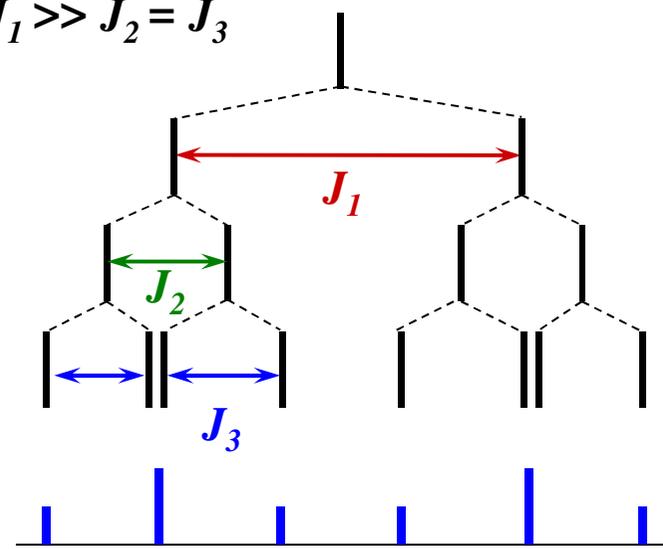
J interaction – multiplet structure

$$J_1 = J_2 = J_3$$



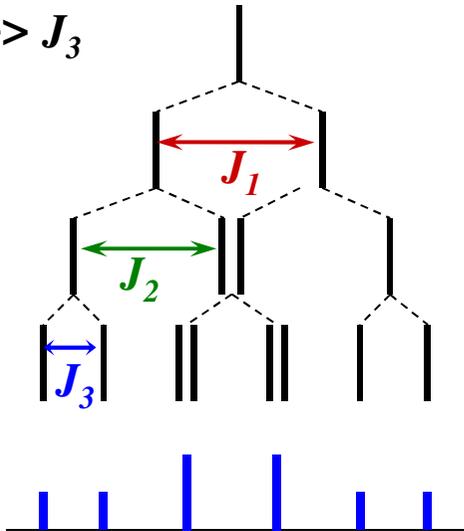
q

$$J_1 \gg J_2 = J_3$$



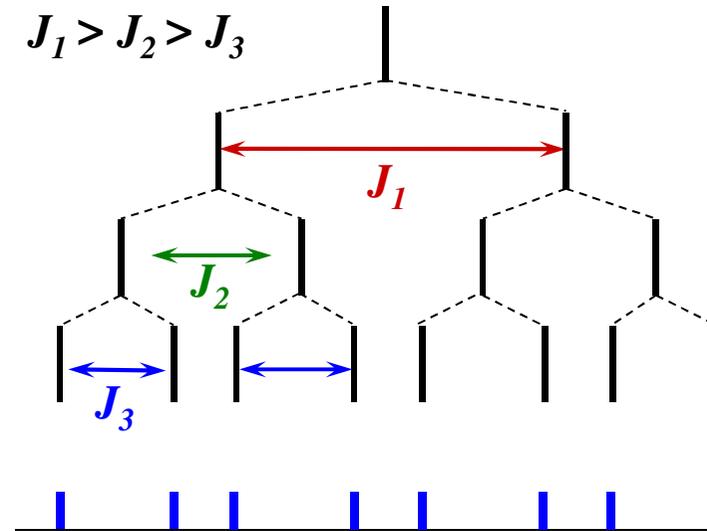
dt

$$J_1 = J_2 \gg J_3$$



td

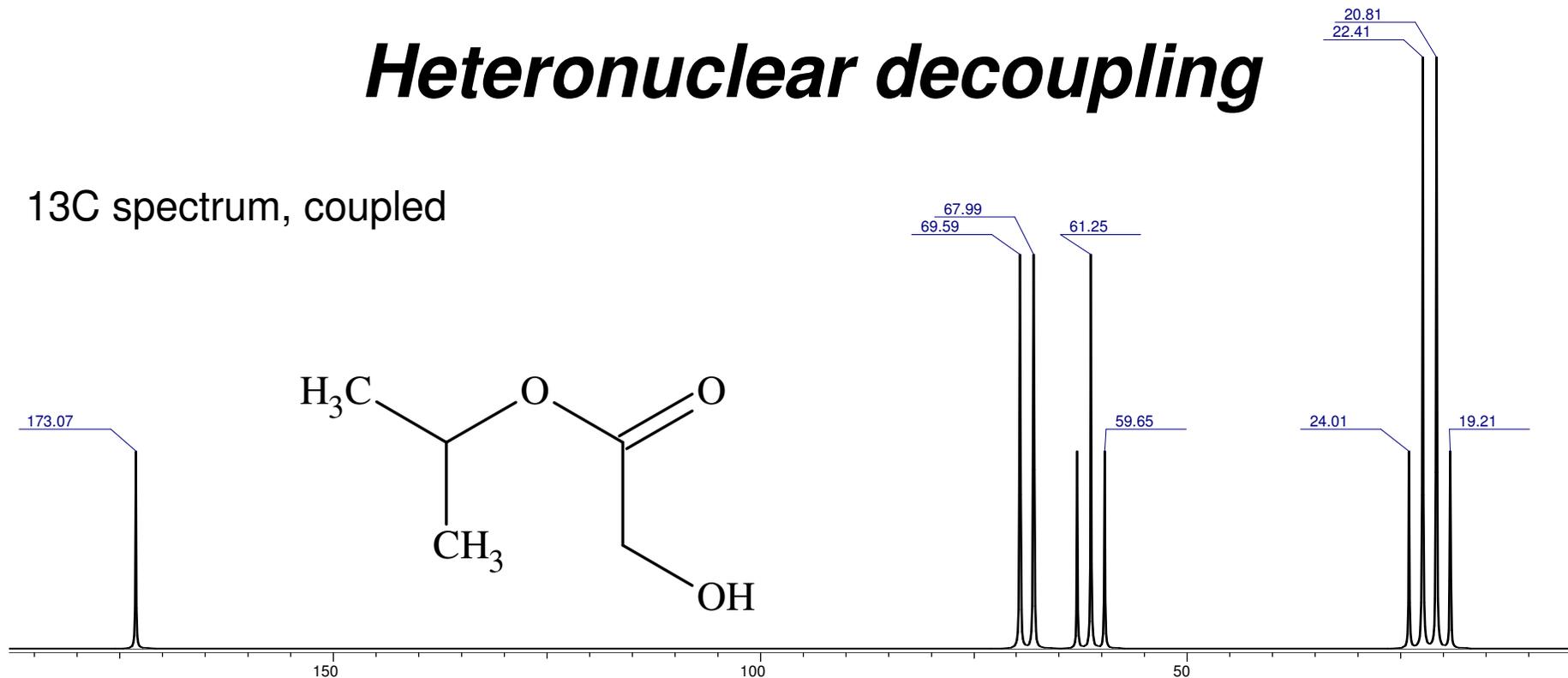
$$J_1 > J_2 > J_3$$



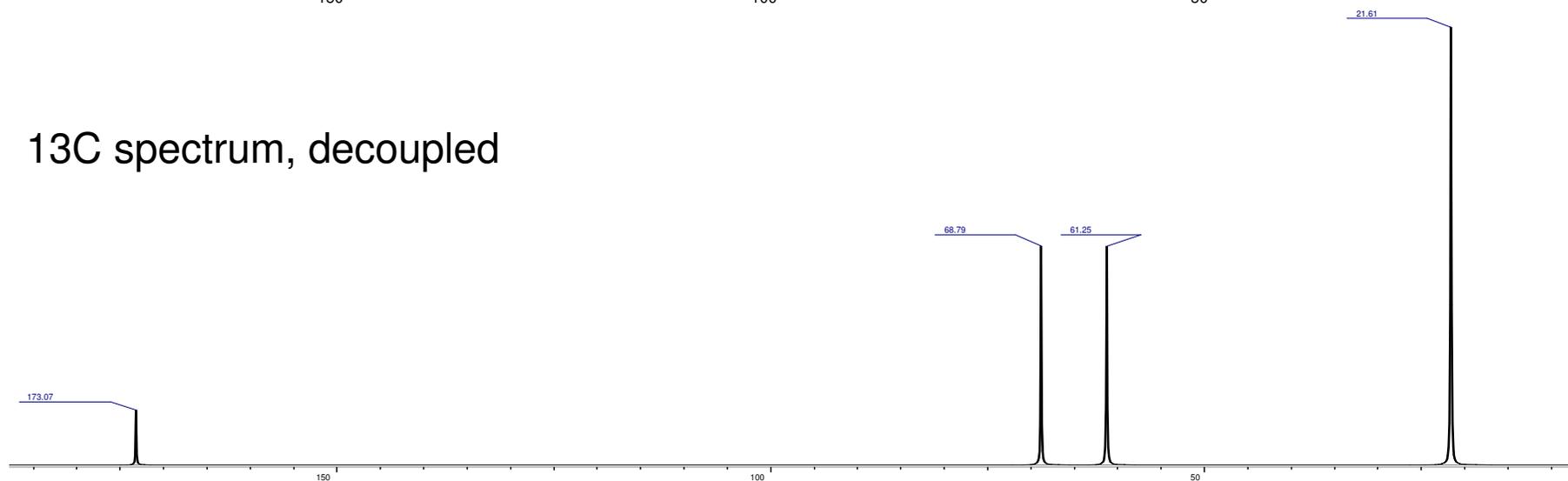
ddd

Heteronuclear decoupling

^{13}C spectrum, coupled



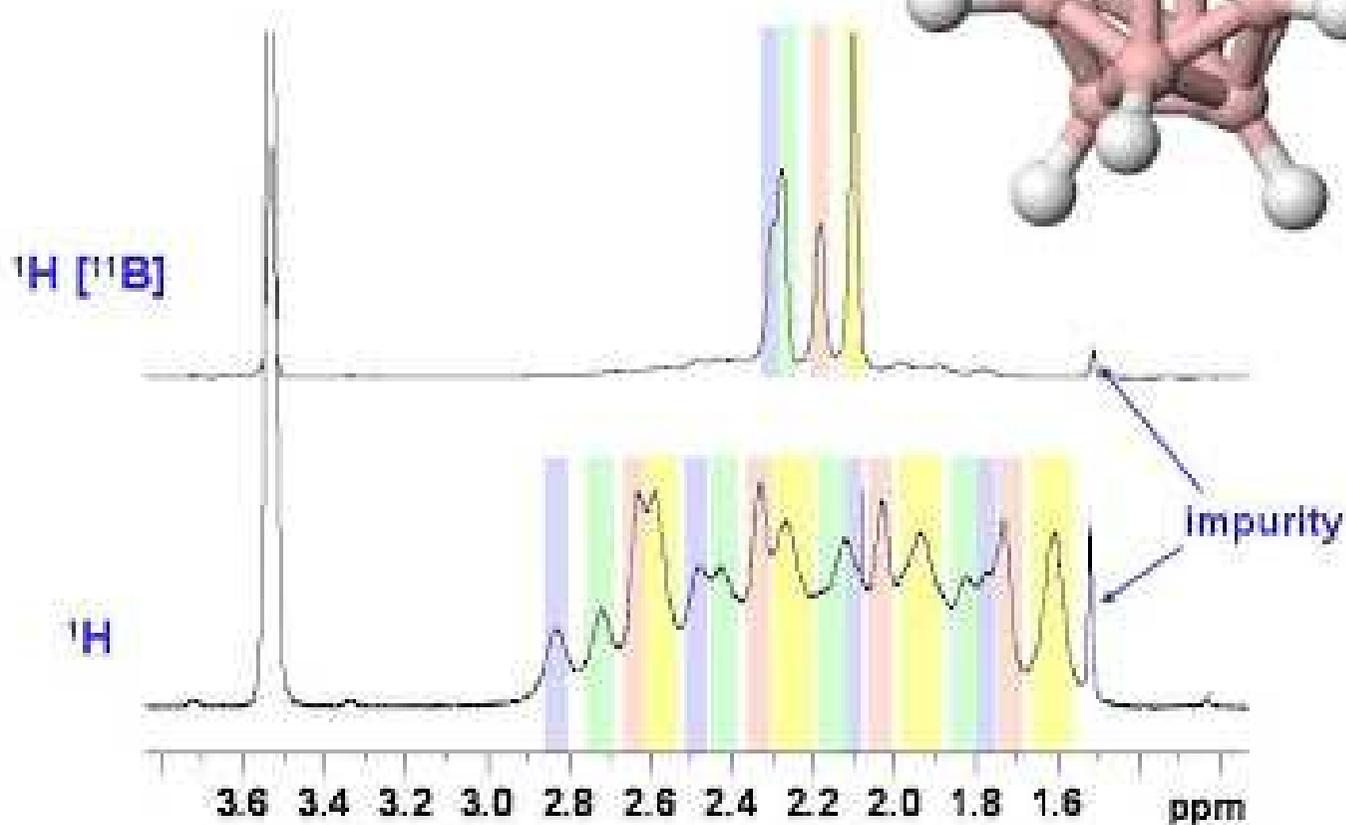
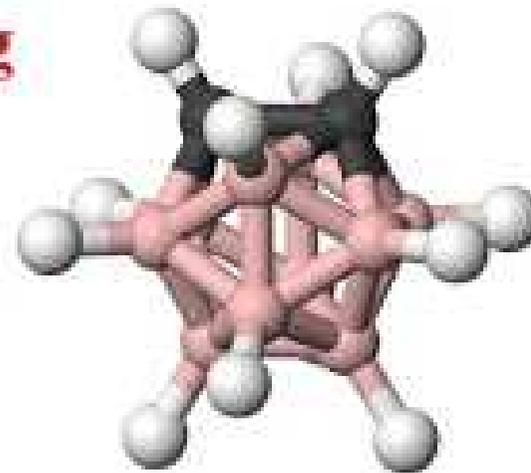
^{13}C spectrum, decoupled



Heteronuclear decoupling

^1H NMR with ^{11}B Decoupling

ortho-Carborane ($\text{C}_2\text{B}_{10}\text{H}_{12}$)



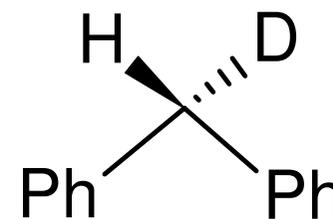
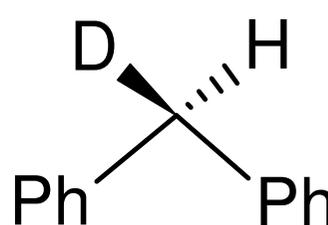
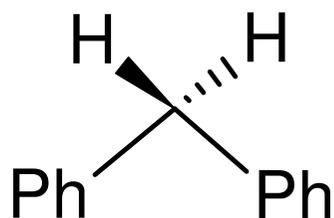
Exercise

Draw ^1H and ^{13}C (both coupled and decoupled) spectra of ethanol

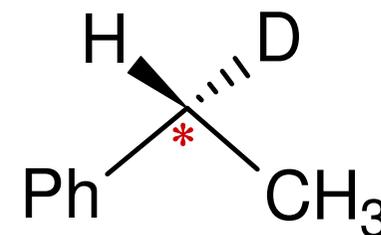
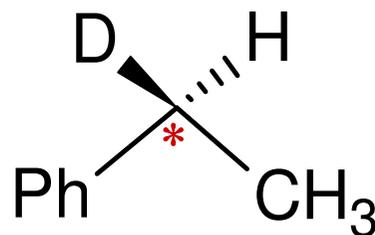
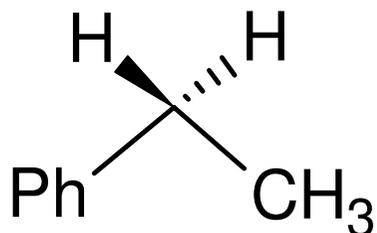
Chemical equivalence

- Nuclei are chemically equivalent if there is asymmetry operation that connects them
- CH₃ protons are equivalent due to fast rotation
- Chemically equivalent nuclei have the same chemical shift

Homotopic

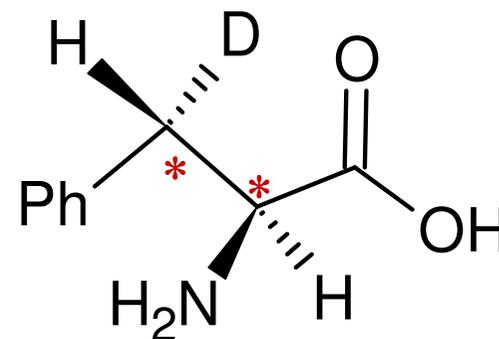
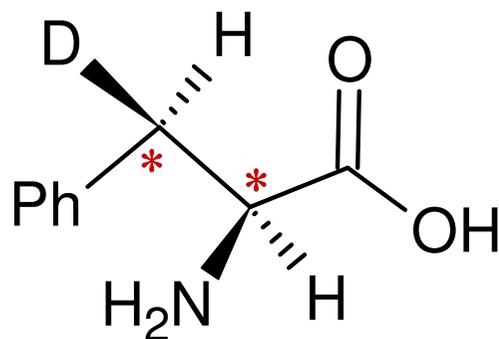
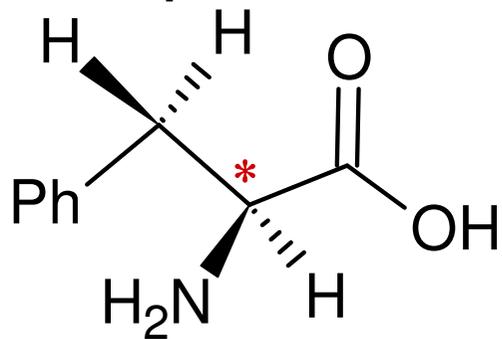


Enantiotopic

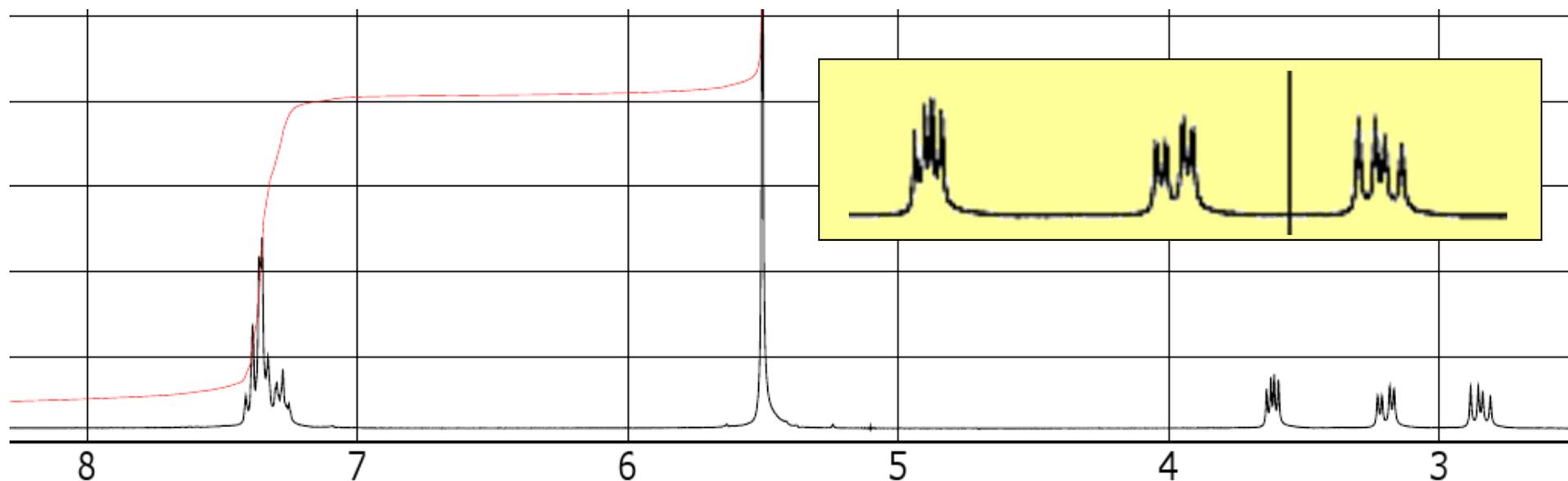


Chemical equivalence

Diastereotopic



NOT chemically equivalent

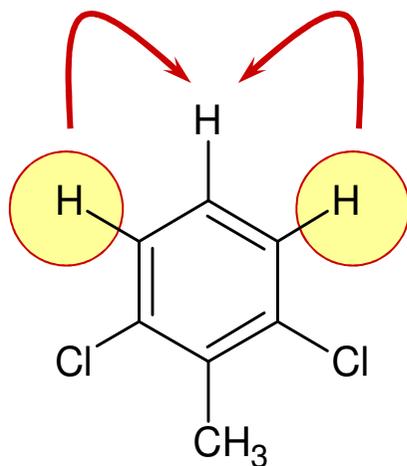


Magnetic equivalence

Nuclei are chemically equivalent

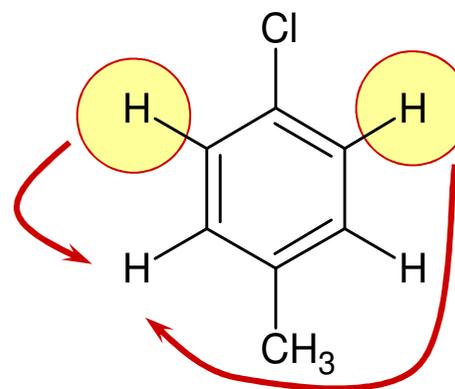
AND

Have the same geometric relation to all other NMR active nuclei
(have the same J interactions with all other NMR active nuclei)



Yes

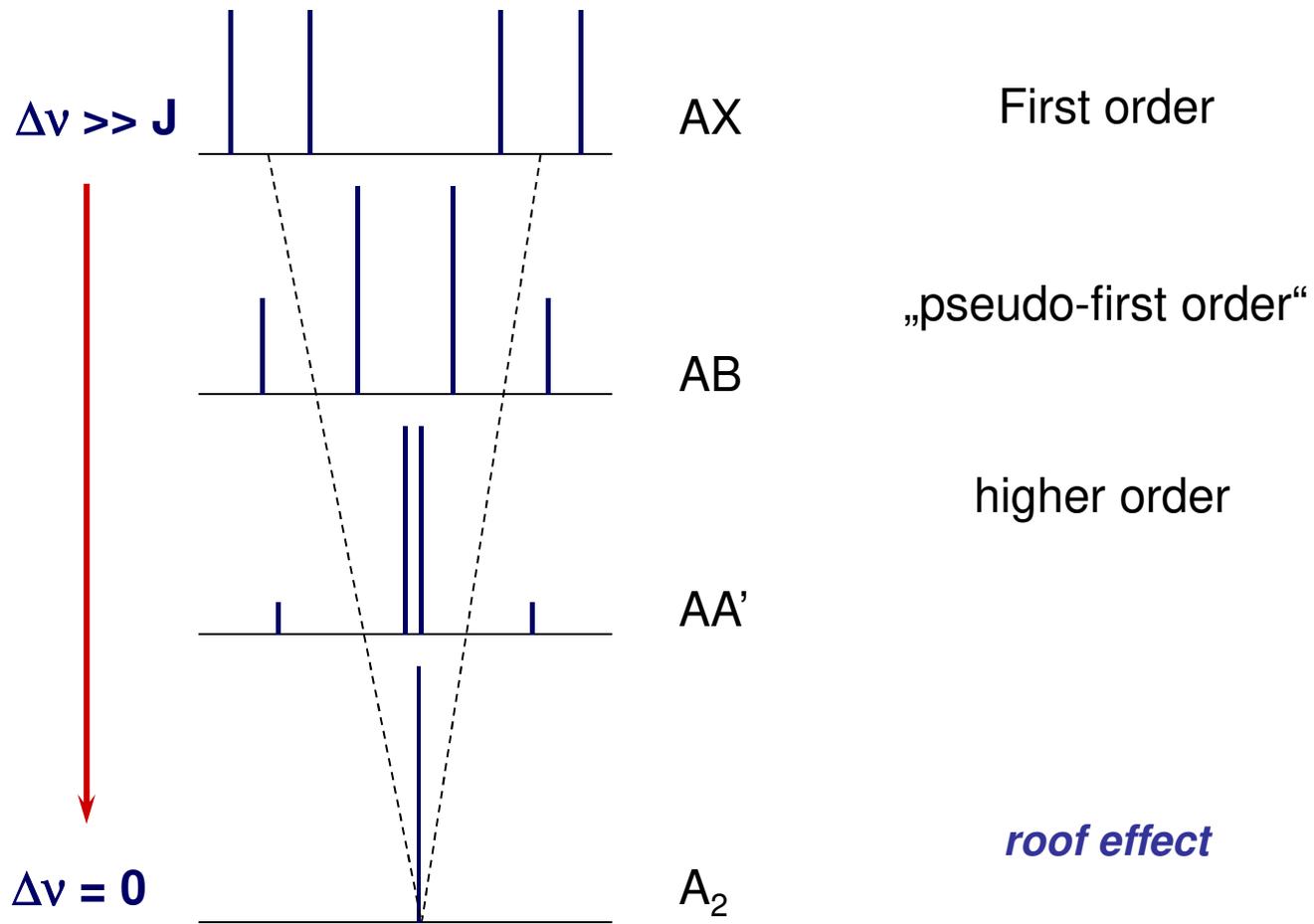
*Spectra of magnetic equivalent nuclei
are not influenced by J-couplings
between them*



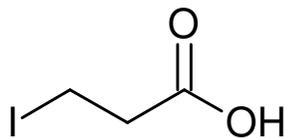
No

*Chemically equivalent but magnetically non-
equivalent nuclei provide spectra of higher
order (complicated pattern due to J-
couplings)*

Higher order spectra

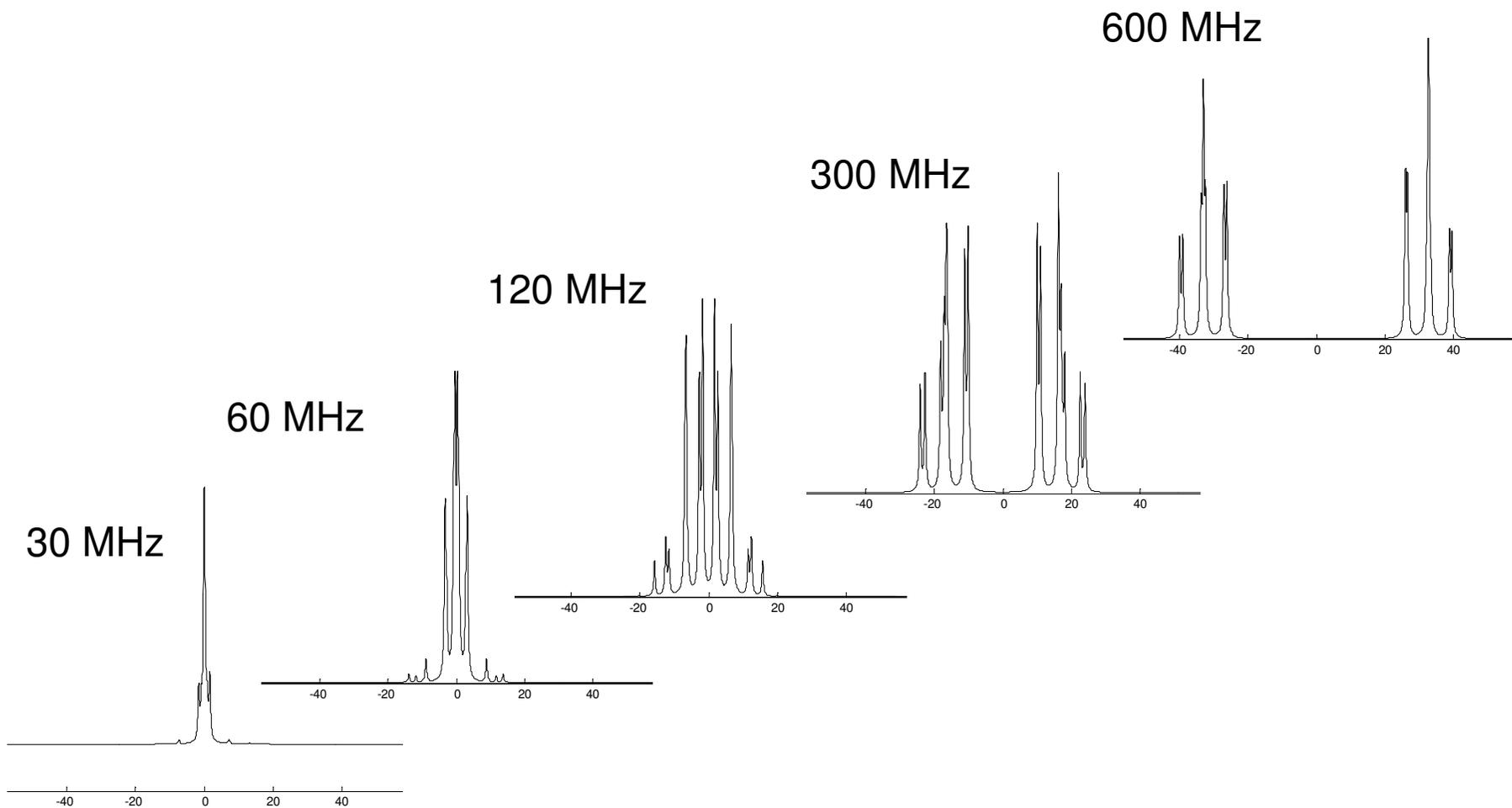


Higher order spectra and mag. field



Chemical shift difference 0.11 ppm

J-coupling 7 Hz



Information from ^1H spectrum

- Number of signals ***symmetry***
- Intensity ***number of equivalent nuclei***
- Chemical shift ***functional group***
- Fine structure and J-constants ***neighboring protons***

Information from ^{13}C spectrum (decoupled)

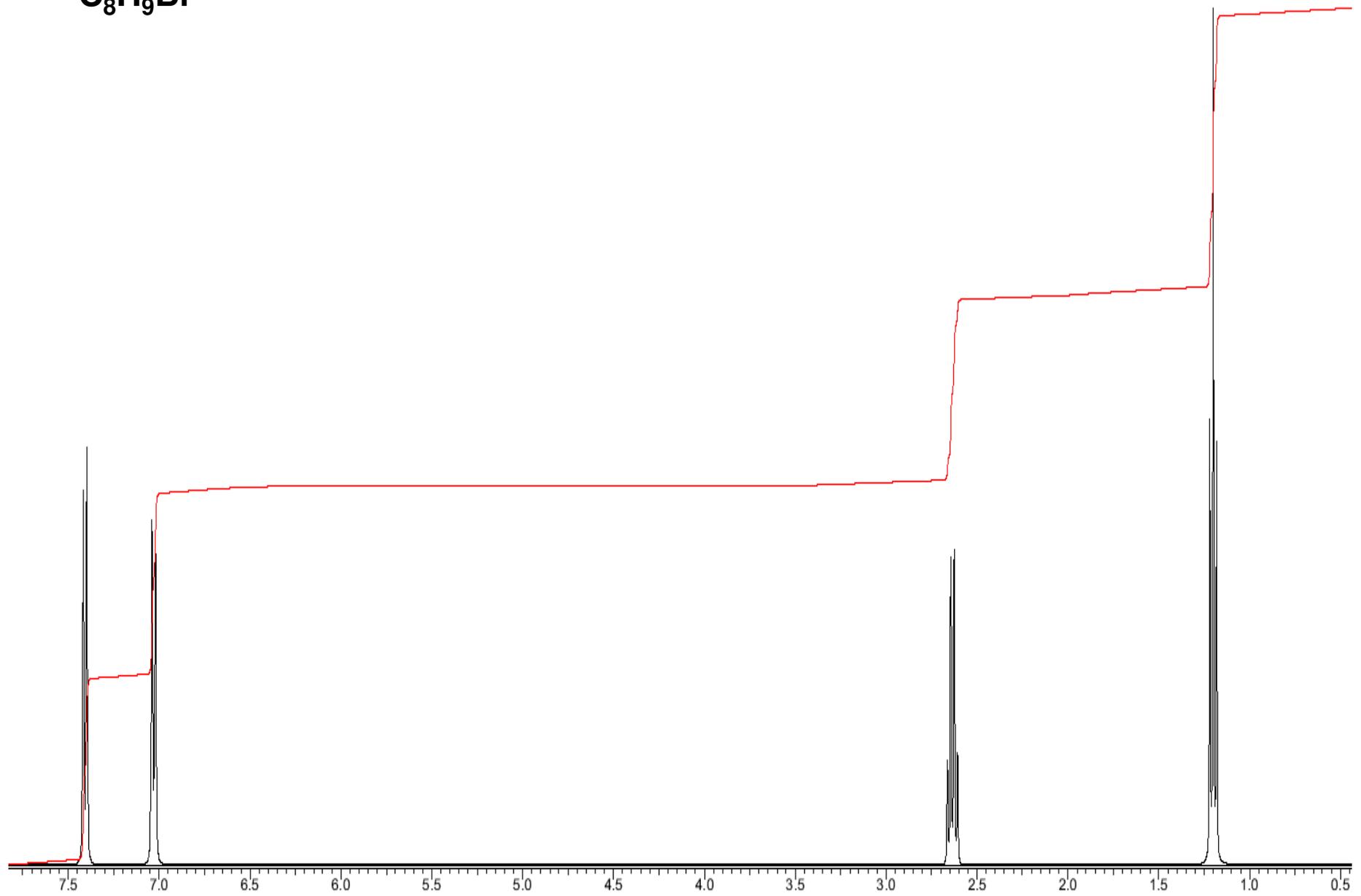
- Number of signals
- Chemical shift
- intensity does not correspond to number of equivalent nuclei

Solving NMR spectra

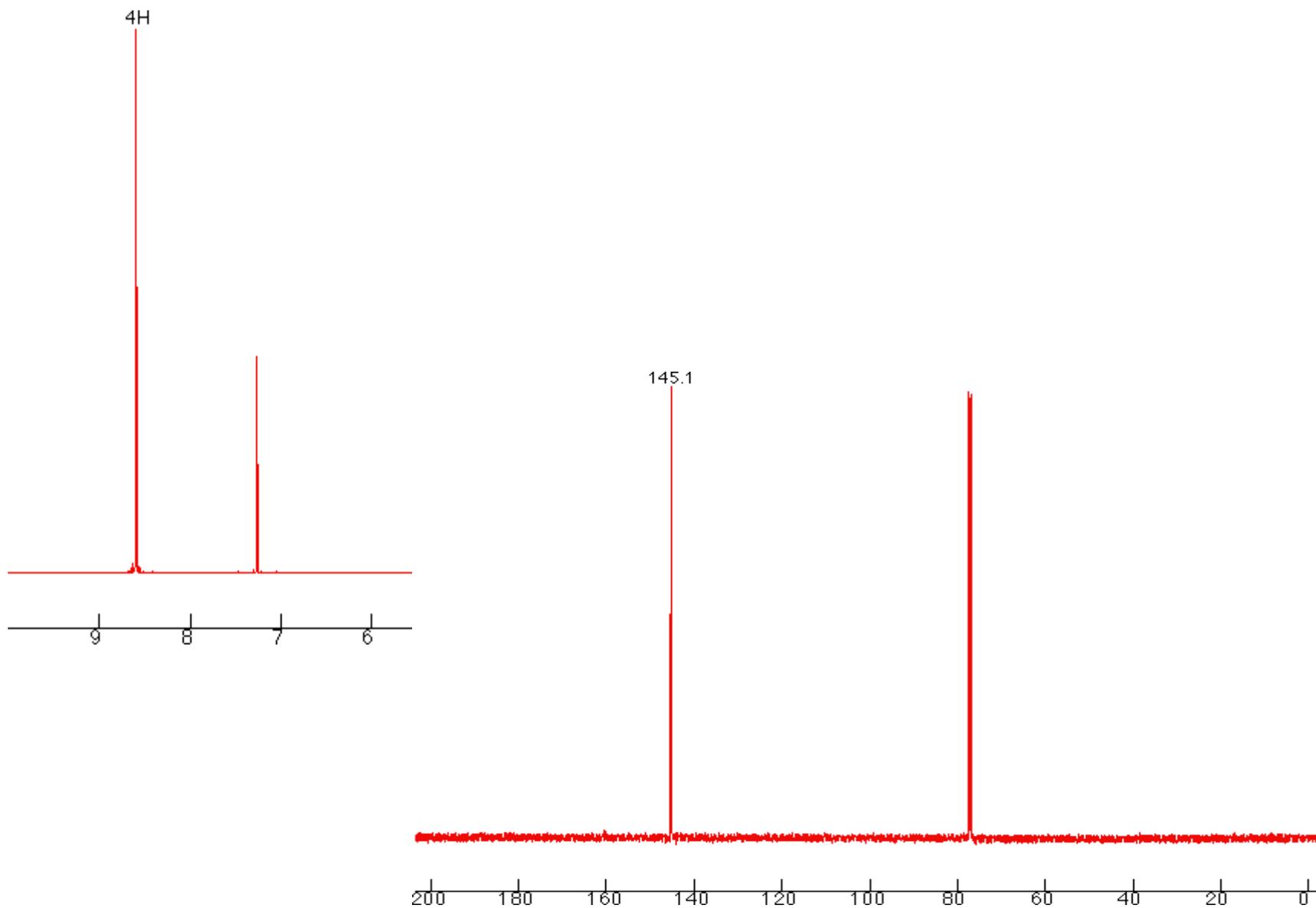
- Read number of signals, their position (shift) and intensity in ^1H spectrum
- split protons to corresponding groups
- from splitting patterns decide which groups are neighbors
- guess which functional groups they are according to chem. shifts (and perhaps using additional info like ^{13}C spectrum, APT/DEPT)
- draw possible molecule
- check what spectrum that molecule would give to verify the solution

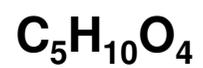
C_8H_9Br

P4

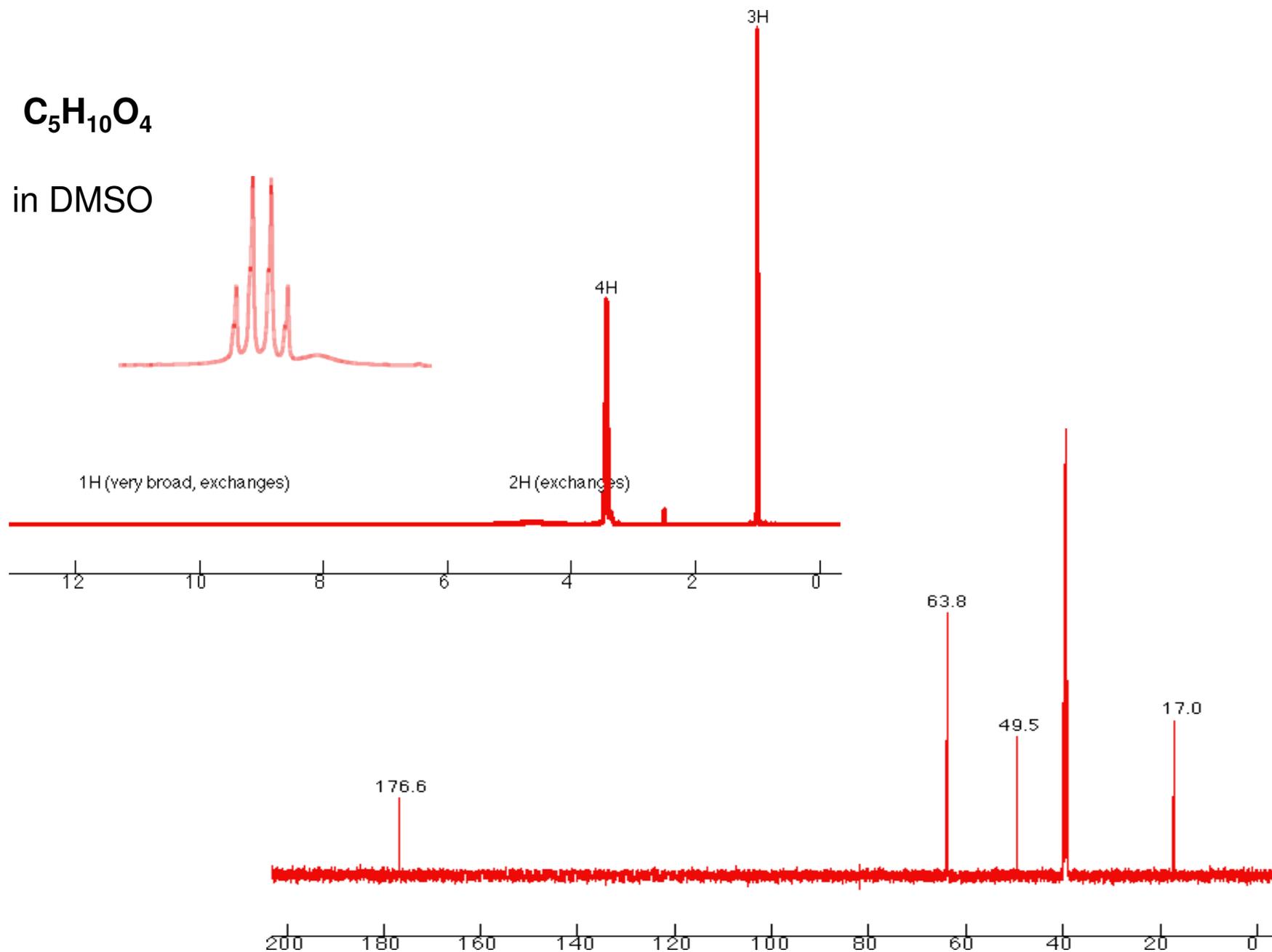


$C_4H_4N_2$ in $CDCl_3$



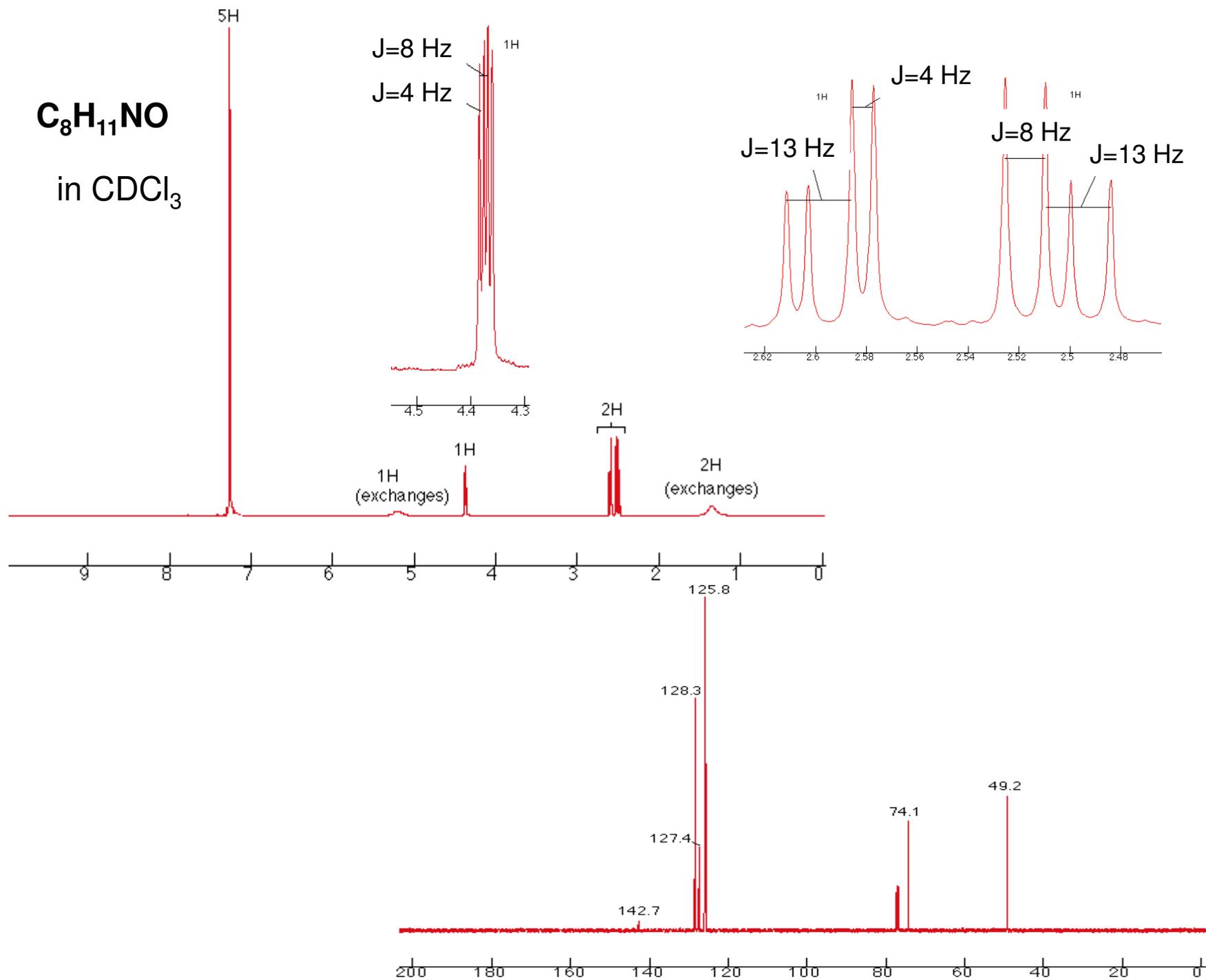


in DMSO





in CDCl₃





in CDCl₃

