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A hypoplastic model for clays improved for undrained conditions

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ABSTRACT: A shortcoming of the hypoplastic model for clays proposed by the first author is an incorrect prediction of the initial portion of the undrained stress path, particularly for tests on normally consolidated soils. A conceptually simple modification of this model, which overcomes this drawback, is proposed in the contribution. The modified model is applicable to both normally consolidated and overconsolidated soils and predicts the same swept-out-memory states as the original model. Under K_0 conditions the modified model gives similar predictions as the original model.

1 Introduction

Soil behaviour is markedly non-linear outside the very small strain range and it is now recognized that this non-linearity must be captured by constitutive models. Non-linear behaviour of overconsolidated soils may be incorporated into standard elasto-plastic models by different means, such as kinematic hardening plasticity, bounding surface plasticity, generalised plasticity and other. These models allow us to predict soil behaviour with good accuracy. It may be, however, argued that the incorporation of non-linearity is often payed by a too complex mathematical formulation or a large number of material parameters, which restrict the models to become more widely used in the engineering practice.

A different approach to model the soil non-linearity is the theory of hypoplasticity (e.g., Kolymbas, 1991). The hypoplastic models are characterised by a single equation nonlinear in stretching $\mathbf{D}$ and they are thus capable of predicting the non-linear behaviour of geomaterials in a straightforward and natural way. The rate formulation of an advanced version of hypoplastic models (e.g., Gudehus, 1996) reads

$$\overset{\circ}{\mathbf{T}} = f_s \boldsymbol{\mathcal{L}} : \mathbf{D} + f_s f_d \mathbf{N} \|\mathbf{D}\| \tag{1}$$

where $\mathcal{L}$ and $\mathbf{N}$ are fourth- and second-order constitutive tensors, which are functions of the normalised stress $\hat{\mathbf{T}}=\mathbf{T}/\mathrm{tr}\mathbf{T}$, the scalar *barotropy* factor f_s is a function of mean stress *p* and the *pyknotropy* factor f_d depends on mean stress *p* and void ratio *e*.

The form of Eq. (1) is advantageous due to its simplicity, but poses restrictions on the model development. As outlined by Niemunis (2002) and recently by Huang et al. (2006), one of the limitations of Eq. (1) is that at normally consolidated and slightly overconsolidated states the hypoplastic models do not perform correctly in predicting the behaviour under undrained conditions. The significant translation of the elliptical response envelope needed to simulate soft behaviour upon isotropic loading and stiff behaviour upon unloading leads to the development of excessive pore pressures in undrained compression and extension close to the isotropic stress state (see response envelope of the original model in Fig. 2 and predicted undrained stress paths in Fig. 4a). Niemunis (2002) and Huang et al. (2006) proposed modifications of hypoplastic models to overcome this shortcoming, which are similar from the conceptual point of view. In both cases the tensor $\mathcal{L}$ is made bi-linear in $\mathbf{D}$ such that

$$\overset{\circ}{\mathbf{T}} = \mathcal{L}^D : \mathbf{D} + \mathbf{N}\|\mathbf{D}\| \tag{2}$$

where $\mathcal{L}^D$ is given by

$$\mathcal{L}^D = \begin{cases} \mathcal{L}_1 & \text{for} \quad \hat{\mathbf{T}} : \mathbf{D} > 0 \\ \mathcal{L}_2 & \text{for} \quad \hat{\mathbf{T}} : \mathbf{D} \leq 0 \end{cases} \tag{3}$$

By enforcing $\mathbf{N}=\mathbf{0}$ at the isotropic stress state, the initial slope of the undrained stress path is perpendicular to the *p* axis, which better represents the measured soil behaviour. Note that $\mathbf{N}=\mathbf{0}$ at the isotropic stress state does not imply hypoelastic response. Rather, as $\mathcal{L}^D$ is bi-linear in $\mathbf{D}$, the predictions correspond to an elasto-plastic model with different tangent stiffnesses in loading ($\mathcal{L}_2$) and unloading ($\mathcal{L}_1$).

The model by Huang et al. (2006) has been developed with the particular aim to predict the behaviour of normally consolidated fine-grained soils. The model is characterised by five soil parameters equivalent to parameters of the model by Mašín (2005) and in comparison with this model a clear improvement in predictions under undrained conditions and equivalent and acceptable predictions of drained behaviour have been demonstrated.

Limitation of the model by Huang et. al stems from the fact that the *pyknotropy* factor f_d from (1) has been omitted and the model thus does not take into account the influence of void ratio (overconsolidation ratio, *OCR*) on soil behaviour. Moreover, the critical state conditions are characterised by a unique state locus in the stress space only. The positions of the critical state lines in the ln *p* vs. ln (1+*e*) space are different for compression and extension, which contradicts the experimental data (see, e.g., the data on Bothkennar Clay by Allman & Atkinson (1992) in Fig. 7c and compare with predictions by the model by Huang et al.). Moreover, the slope of the critical state line in ln *p* vs. ln (1+*e*) space, as well as the slopes of normal compression lines for $\eta=q/p\neq 0$, are not unique, whereas it is commonly accepted that the slopes of the isotropic and K_0 normal compression lines and the critical state line are the same.

In this contribution, a simple modification of the hypoplastic model for clays by Mašín (2005) (denoted as *original* model), which does not suffer from the shortcomings outlined above, is presented. The approach employed may be seen as a further development and application of the work by Niemunis (2002). In the paper, continuum mechanics sign convention (compression negative) is adopted throughout, except for Roscoe stress variables *p* and *q*, which are positive in compression. Stretching and stress tensors are restricted to triaxial states, with $\mathbf{D}=[(D_a,0,0),(0,D_r,0),(0,0,D_r)]$ and $\mathbf{T}=[(T_a,0,0),(0,T_r,0),(0,0,T_r)]$. For the details of notation used in this contribution, see Mašín (2005), for more detailed description and evaluation of the modified model the reader is referred to Mašín & Herle (2007).

2 Modification of the model

For details of the formulation of the original hypoplastic model for clays the reader is referred to Mašín (2005). The enhanced model (denoted here as *modified* model) is characterised by the following rate form:

$$\overset{\circ}{\mathbf{T}} = f_s \mathcal{L} : \mathbf{D} + f_s f_d \mathbf{N} D_n \tag{4}$$

where $D_n=\|\mathbf{D}\|$ of the basic hypoplastic model is replaced by

$$D_n = w_y \|\mathbf{D}\| + (1 - w_y) \left| \vec{\mathbf{D}}_{SOM} : \mathbf{D} \right| \tag{5}$$

The weighting factor w_y has been defined by Niemunis (2002) as

$$w_y = \begin{cases} \left(\dfrac{Y - Y_i}{1 - Y_i}\right)^{\xi} & \text{for} \quad Y \leq 1 \\ 1 & \text{for} \quad Y > 1 \end{cases} \tag{6}$$

where the quantity Y, which is part of the mathematical formulation of the original model, has the following properties:

$$\begin{array}{ll} Y = Y_i < 1 & \text{at the isotropic stress state} \\ Y = 1 & \text{at the critical stress state} \\ Y > 1 & \text{for } \varphi_{mob} \text{ higher than the critical stress state} \end{array} \tag{7}$$

Therefore, w_y=1 at the critical stress state and for higher mobilised friction angles ($\varphi_{mob} \geq \varphi_c$), w_y=0 at the isotropic stress state and $0<w_y<1$ for $Y_i<Y<1$. The exponential interpolation is controlled by a new model parameter ξ. Eq. (6) is graphically represented in Fig. 1.

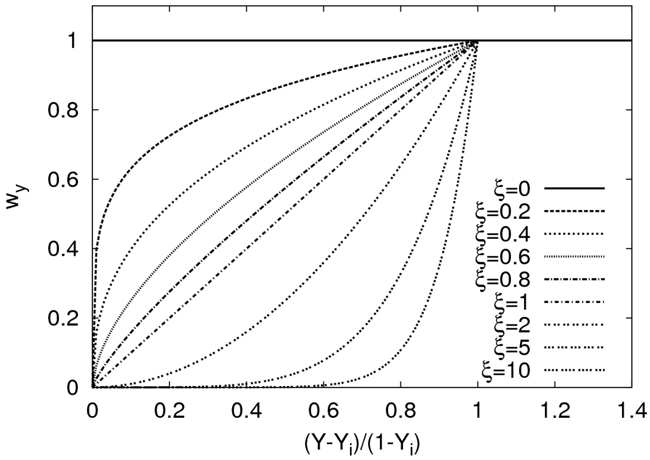


Figure 1. Influence of the new parameter ξ on the weighting factor w_y (6).

$\vec{\mathbf{D}}_{SOM}$ in (5) is the direction of stretching at swept-out-memory conditions that correspond to the current stress state $\hat{\mathbf{T}}$, calculated using a procedure developed in Mašín & Herle (2005):

$$\vec{\mathbf{D}}_{SOM} = -\frac{\boldsymbol{\mathcal{A}}^{-1} : \mathbf{N}}{\|\boldsymbol{\mathcal{A}}^{-1} : \mathbf{N}\|} \tag{8}$$

where the fourth-order tensor $\boldsymbol{\mathcal{A}}$ is defined as

$$\boldsymbol{\mathcal{A}} = f s \boldsymbol{\mathcal{L}} + \frac{1}{\lambda^*}\mathbf{T} \otimes \mathbf{1} \tag{9}$$

with λ^* being a parameter of the original model. The proposed formulation has the following properties:

1. At the isotropic stress state Y=Y_i, therefore w_y=0 and because $\vec{\mathbf{D}}_{SOM}$=$\vec{\mathbf{1}}$, D_n=|$\vec{\mathbf{1}}$:$\mathbf{D}$|. For the isotropic loading and unloading $\vec{\mathbf{D}}$=$\mathbf{D}$/‖$\mathbf{D}$‖=$\vec{\mathbf{1}}$, therefore D_n=‖$\mathbf{D}$‖ and predictions by the original model are recovered. For undrained loading (tr$\mathbf{D}$=0), however, D_n=0, so predictions are controlled by the linear part $f_s\boldsymbol{\mathcal{L}}$:$\mathbf{D}$ only and since the tensor $\boldsymbol{\mathcal{L}}$ is isotropic,

undrained stress path is initially perpendicular to the *p*=axis. In other words, hypoelastic response envelope defined by $f_s\mathcal{L}$ is squeezed for $\mathrm{tr}\mathbf{D}<0$, elongated for $\mathrm{tr}\mathbf{D}>0$ and unchanged for $\mathrm{tr}\mathbf{D}=0$. For equivalent parameters it coincides with the response envelope of the model by Huang et. al. The response envelopes of the original and modified models for normally consolidated soil at the isotropic stress state are shown in Fig. 2. The parameters for London Clay from Mašín (2005) (Tab. 1) are used throughout this paper (Figs. 2-4), with ξ=0.6 when demonstrating predictions by the modified model.

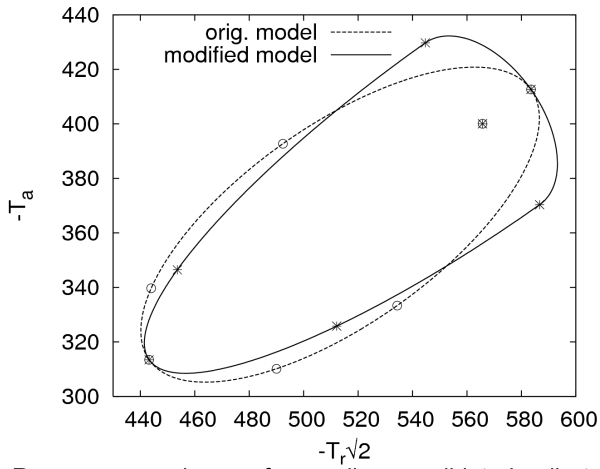


Figure 2. Response envelopes of normally consolidated soil at the isotropic stress state predicted by the original and modified models. The symbols in the above figure stand for isotropic loading and unloading, undrained compression and extension and states with $D_a=0$ and $D_r=0$ for $\mathrm{tr}\mathbf{D}>0$.

2. At the critical stress state and higher φ_{mob} we get $Y\geq 1$, so $w_y=1$ and $D_n=\|\mathbf{D}\|$ for any direction of stretching. In this case the predictions by the original and modified model coincide.
3. At swept-out-memory conditions (defined in the stress vs. void ratio space) $\vec{\mathbf{D}}=\vec{\mathbf{D}}_{SOM}$, therefore $|\vec{\mathbf{D}}_{SOM}:\mathbf{D}|=\|\mathbf{D}\|$ and $D_n=\|\mathbf{D}\|$ for any value of w_y. The predictions by the original and modified models coincide.
4. For other cases the terms $\|\mathbf{D}\|$ and $|\vec{\mathbf{D}}_{SOM}:\mathbf{D}|$ in (5) are weighted by w_y, the relative importance of both terms is controlled by the new parameter ξ.

Table 1. Parameters of the original model used in this paper.

Soil	φ_c [°]	λ^*	κ^*	N	r
London Clay	22.6	0.11	0.016	1.375	0.4
Fujinomori Clay	34	0.0445	0.0108	0.8867	1.3
Bothkennar Clay	35	0.119	0.006	1.344	0.07

Thanks to the condition 3 the modified model predicts the same swept-out-memory surface (Mašín & Herle, 2005) (and, therefore, normal compression lines in the $\mathbf{T}$ vs. *e* space) as the original model. The parameter ξ controls the shape of the undrained stress path. Its influence on undrained stress path for a normally consolidated clay starting from an isotropic stress state is demonstrated in Fig. 3.

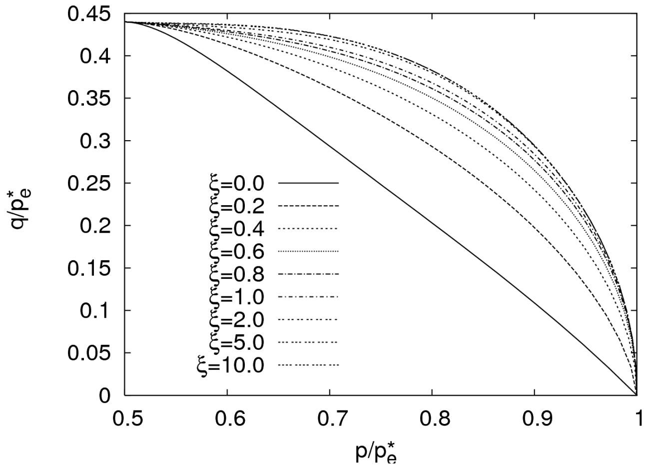
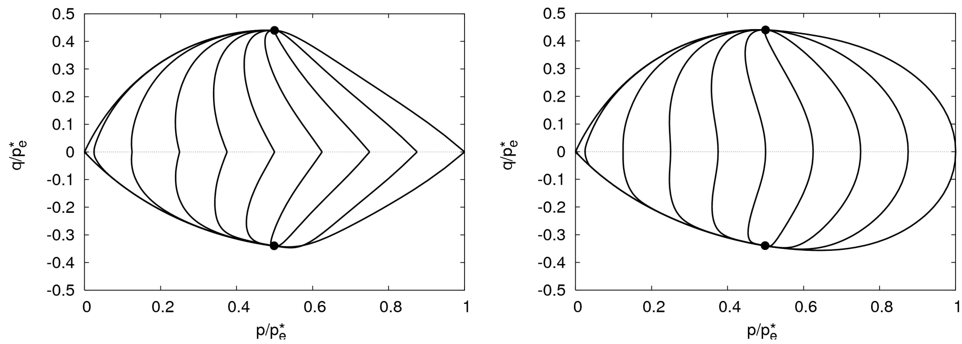


Figure 3. The influence of the parameter ξ on the shape of the undrained stress path of normally consolidated soil starting from the isotropic stress state. p_e* is the Hvorslev equivalent pressure on the isotropic normal compression line

Figure 4 shows undrained stress paths predicted by the original (a) and modified (b) models for compression and extension tests starting at the isotropic stress state for different *OCRs*. Clearly, the shapes of the stress paths on normally consolidated and slightly overconsolidated soils are more realistic, while the model still correctly captures the qualitative influence of overconsolidation.



(a) (b)

Figure 4. Stress paths of undrained compression and extension tests with different initial values of *OCRs*. The original model (a) and the modified model with ξ=0.6 (b).

Mašín & Herle (2007) further demonstrated that Eqs. (4) and (5) may be re-written in the form similar to Eq. (2). The response envelopes of the modified model are thus similarly to the model by Huang et al. composed of two elliptic sections, their centres are translated with respect to the reference stress state at anisotropic stress states. They have also shown that the proposed modification does not have a significant impact on model predictions for anisotropic stress states and for higher *OCRs*.

3 Evaluation of the model

3.1 Normally consolidated soil at the isotropic stress state

In this section the model will be evaluated with respect to experimental data on normally consolidated Fujinomori Clay by Nakai et al. (1986). For details on calibration of the original hypoplastic model and the model by Huang et al. using drained compression test the reader is referred to Huang et al. (2006). The parameters of the original model are given in Tab. 1. It may be seen from Fig. 5 that the parameters of the original model evaluated by Huang et al. (2006) are suitable for predicting the drained test also with the modified model by assuming the new parameter ξ=0.6. In fact, the predictions by the original and modified models along drained stress path are very similar and reproduce well the observed soil behaviour. The only discrepancy is that the compressive volumetric strains are underpredicted by the original and modified models. This is due to a fixed position of the isotropic normal compression line with respect to the critical state line in the ln *p* vs. ln(1+*e*) space, which could be varied by a slight modification of the original model.

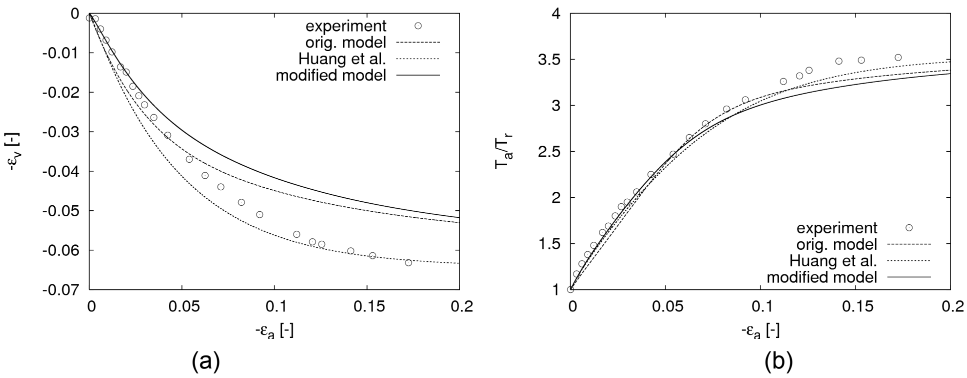


Figure 5. Drained compression test from the isotropic normally consolidated state on Fujinomori Clay (Nakai et. al, 1986), predictions by the original, modified and Huang et al. Models

The parameters from Tab. 1 were used for simulation of undrained compression and extension tests on Fujinomori Clay (Fig. 6). It is clear that the predictions of the modified model for tests on normally consolidated soils starting from the *isotropic* stress states are significantly different as compared to the original model. With parameters evaluated using drained tests the modified model captures well the soil behaviour under undrained stress paths, and its predictions are very similar to the model by Huang et al.

3.2 Anisotropic stress state

Figure 7 shows the experimental results of undrained compression and extension experiments on reconstituted Bothkennar Clay by Allman & Atkinson (1992). The specimens were under K_0 conditions normally consolidated and then unloaded to four different *OCRs*. The simulations by the original and modified models with parameters evaluated in Mašín (2007) with slightly increased value of the parameter κ^* (Tab. 1) and ξ=0.6 are shown in Figs. 7 (a) and (b). The figures also include the swept-out-memory surfaces predicted by the original and modified models. It is clear that at K_0 initial conditions both models yield similar results, as discussed in more detail by Mašín & Herle (2007). The shape of the undrained stress paths may be considered as satisfactory, at least

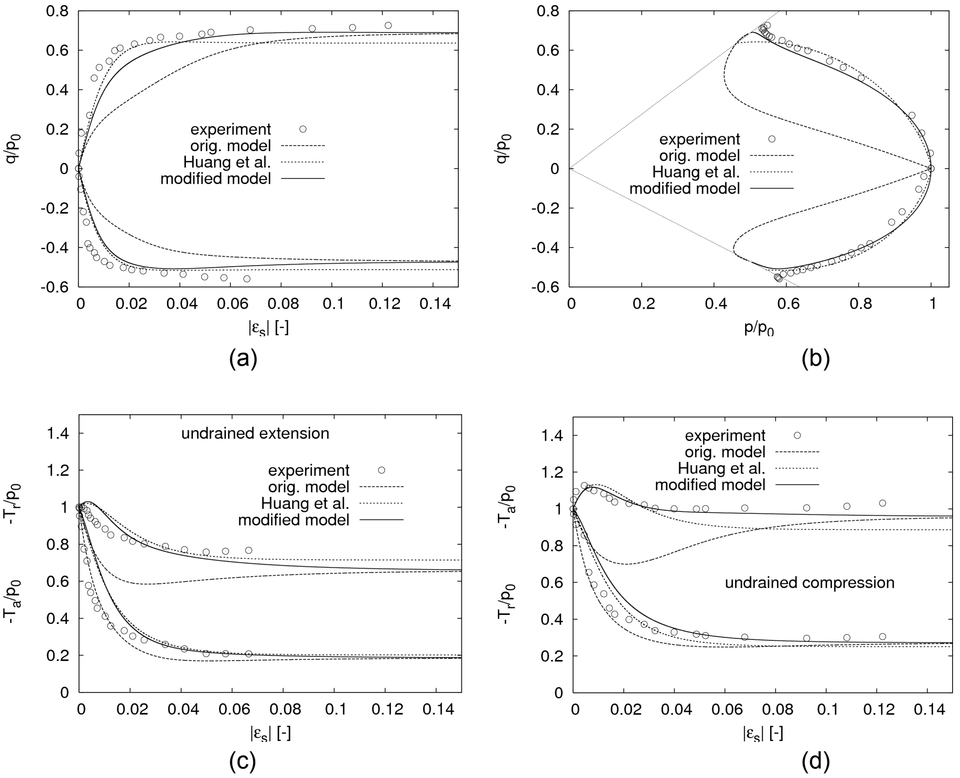


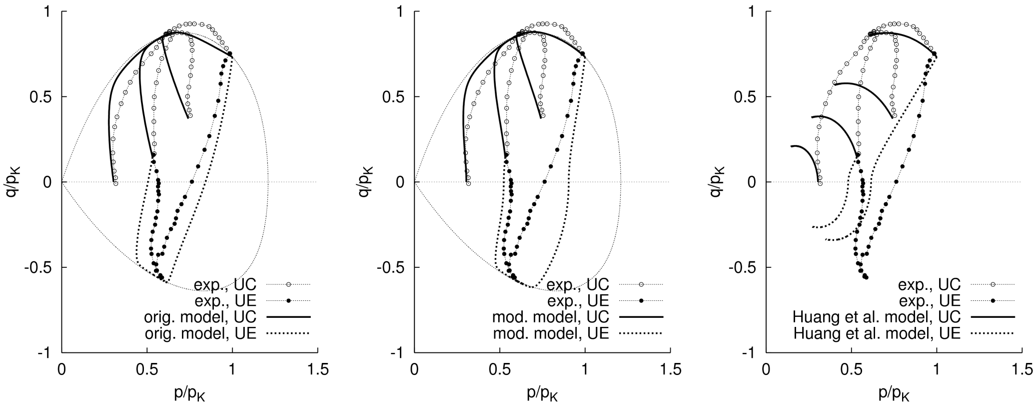
Figure 6. Undrained compression and extension tests from the isotropic normally consolidated state on Fujinomori Clay (Nakai et al., 1986), predictions by the original and modified models and model by Huang et al. p_0 is the initial mean stress.

from the qualitative point of view. For comparison, Fig. 7 (c) shows predictions by the model by Huang et al. The predictions are similar with predictions by the original and modified models for undrained compression from the K_0 normally compressed state only. Fig. 7 (c) clearly demonstrates two important shortcomings of the model by Huang et al. - the positions of the critical state lines are different for compression and for extension and the influence of *OCR* is not taken into account.

4 Concluding remarks

This contribution presents a conceptually simple modification of the hypoplastic model for clays by Mašín (2005). The modification is aimed to improve performance of the model at low *OCRs*, particularly to improve predictions under undrained conditions for isotropically consolidated specimens. At normally consolidated states, the model yields similar results as a recently proposed hypoplastic model by Huang et al. (2006), however, the modified model can also be used to predict the behaviour of overconsolidated soil. Also, the proposed modification has no negative impact on the predictions of the swept-out-memory states by the original model.

It should be pointed out that the modified model does not change significantly predictions by the original model for specimens with K_0 initial states. As the K_0 states are present in the ground, the proposed modification cannot be expected to have significant impact on practical application of the hypoplastic model by Mašín (2005). Rather, the modification can be regarded as an inspection into some properties of hypoplastic models in general.



(a) (b) (c)

Figure 7. Undrained compression and extension experiments from K_0 states on Bothkennar Clay (Allman & Atkinson, 1992), stress paths normalised with respect to equivalent pressure on the K_0 normal compression line. Predictions by the original (a), modified (b) and Huang et al. (c) models. p_K is the equivalent mean stress at the K_0 normal compression line.

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