Improvement of a hypoplastic model to predict clay behaviour under undrained conditions

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ABSTRACT

A shortcoming of the hypoplastic model for clays proposed by the first author is an incorrect prediction of the initial portion of the undrained stress path, particularly for tests on normally consolidated soils at isotropic stress states. A conceptually simple modification of this model, which overcomes this drawback, is proposed in the contribution. The modified model is applicable to both normally consolidated and overconsolidated soils and predicts the same swept-out-memory states (i.e., normal compression lines) as the original model. At anisotropic stress states and at higher overconsolidation ratios the modified model yields predictions similar to the original model.

1 Introduction

Soil behaviour is outside the very small strain range markedly non-linear and it is now recognized that this non-linearity must be captured by constitutive models. Non-linear behaviour of overconsolidated soils may be incorporated into standard elasto-plastic models [14] by different means, such as kinematic hardening plasticity [10], bounding surface plasticity [3], generalised plasticity [13] and other. These models allow us to predict soil behaviour with good accuracy. It may be, however, argued that the incorporation of non-linearity is often payed by a too complex mathematical formulation or a large number of material parameters, which restrict the models to become more widely used in the engineering practice.

A different approach to model the soil non-linearity is the theory of hypoplasticity [6, 2]. The hypoplastic models are characterised by a single equation nonlinear in stretching $\mathbf{D}$ and they are thus capable of predicting the non-linear behaviour of geomaterials in a straightforward and natural way. The rate formulation of an advanced version of hypoplastic models (e.g., [4]) reads

$$\overset{\circ}{\mathbf{T}} = f_s \boldsymbol{\mathcal{L}} : \mathbf{D} + f_s f_d \mathbf{N} \|\mathbf{D}\| \tag{1}$$

where $\boldsymbol{\mathcal{L}}$ and $\mathbf{N}$ are fourth- and second-order constitutive tensors, which are functions of the normalised stress $\hat{\mathbf{T}} = \mathbf{T}/\operatorname{tr}\mathbf{T}$, the scalar *barotropy* factor f_s is a function of mean stress p and the *pyknotropy* factor f_d depends on mean stress p and void ratio e.

The form of Eq. (1) is advantageous due to its simplicity, but poses restrictions on the model development. As outlined by Niemunis [12] and recently by Huang et al. [5], one of the limitations of Eq. (1) is that at normally consolidated and slightly overconsolidated states the hypoplastic models do not perform correctly in predicting the behaviour under undrained conditions. The significant translation of the elliptical response envelope needed to simulate soft behaviour upon isotropic loading and stiff behaviour upon unloading leads to the development of excessive pore pressures in undrained compression and extension close to the isotropic stress state (see response envelope of the original model in Fig. 2 and predicted undrained stress paths in Fig. 7a).

Niemunis [12] and Huang et al. [5] proposed modifications of hypoplastic models to overcome this shortcoming, which are similar from the conceptual point of view. In both cases the tensor $\boldsymbol{\mathcal{L}}$ is made bi-linear in $\mathbf{D}$ such that

$$\overset{\circ}{\mathbf{T}} = \boldsymbol{\mathcal{L}}^D : \mathbf{D} + \mathbf{N} \|\mathbf{D}\| \tag{2}$$

where $\boldsymbol{\mathcal{L}}^D$ is given by

$$\boldsymbol{\mathcal{L}}^D = \begin{cases} \boldsymbol{\mathcal{L}}_1 & \text{for} \quad \hat{\mathbf{T}} : \mathbf{D} > 0 \\ \boldsymbol{\mathcal{L}}_2 & \text{for} \quad \hat{\mathbf{T}} : \mathbf{D} \leq 0 \end{cases} \tag{3}$$

By enforcing $\mathbf{N} = \mathbf{0}$ at the isotropic stress state, the initial slope of the undrained stress path is perpendicular to the p axis, which better represents the measured soil behaviour. Note that $\mathbf{N} = \mathbf{0}$ at the isotropic stress state does not imply hypoelastic response. Rather, as $\boldsymbol{\mathcal{L}}^D$ is bi-linear in $\mathbf{D}$, the predictions correspond to an elasto-plastic model with different tangent stiffnesses in loading ($\boldsymbol{\mathcal{L}}_2$) and unloading ($\boldsymbol{\mathcal{L}}_1$).

The model by Huang et al. [5] has been developed with the particular aim to predict the behaviour of normally consolidated fine-grained soils. The model is characterised by five soil parameters equivalent to parameters of the model by Mašín [7] and in comparison with this model a clear improvement in predictions under undrained conditions and equivalent and acceptable predictions of drained behaviour have been demonstrated.

Limitation of the model [5] stems from the fact that the pyknotropy factor f_d from (1) has been omitted and the model thus does not take into account the influence of void ratio (overconsolidation ratio, OCR) on soil behaviour. Moreover, the critical state conditions are characterised by a unique state locus in the stress space only. The positions of the critical state lines are in the $\ln p$ vs. $\ln(1+e)$ space different for compression and extension, which contradicts the experimental data (see, e.g., the data on Bothkennar Clay [1] in Fig. 10c and compare with predictions by the model by Huang et al.). Moreover, the slope of the critical state line in $\ln p$ vs. $\ln(1+e)$ space, as well as the slopes of normal compression lines for $\eta = q/p \neq 0$, are not unique, whereas it is commonly accepted that the slopes of the isotropic and K_0 normal compression lines and the critical state line are the same.

In this contribution, a simple modification of the hypoplastic model for clays by Mašín [7] (denoted as *original* model), which does not suffer from the shortcomings outlined above, is developed. The approach employed may be seen as a further development and application of the work by Niemunis [12]. In the paper, continuum mechanics sign convention (compression negative) is adopted throughout, except for Roscoe stress variables p and q, which are positive in compression. Stretching and stress tensors are restricted to triaxial states, with $\mathbf{D} = [(D_a, 0, 0), (0, D_r, 0), (0, 0, D_r)]$ and $\mathbf{T} = [(T_a, 0, 0), (0, T_r, 0), (0, 0, T_r)]$. For the details of notation used in this contribution, see [7].

2 Modification of the model

For details of the formulation of the original hypoplastic model for clays the reader is referred to [7]. The enhanced model (denoted here as *modified* model) is characterised by the following rate form:

$$\mathring{\mathbf{T}} = f_s \boldsymbol{\mathcal{L}} : \mathbf{D} + f_s f_d \mathbf{N} D_n \tag{4}$$

where $D_n = \|\mathbf{D}\|$ of the basic hypoplastic model (1) is replaced by

$$D_n = w_y \|\mathbf{D}\| + (1 - w_y) \, |\vec{\mathbf{D}}_{SOM} : \mathbf{D}| \tag{5}$$

The weight factor w_y has been defined by Niemunis [12] as

$$w_y = \begin{cases} \left(\dfrac{Y - Y_i}{1 - Y_i} \right)^{\xi} & \text{for} \quad Y \leq 1 \\ 1 & \text{for} \quad Y > 1 \end{cases} \tag{6}$$

where the quantity Y, which is part of the mathematical formulation of the original model, has the following properties:

$$\begin{array}{ll} Y = Y_i < 1 & \text{at the isotropic stress state} \\ Y = 1 & \text{at the critical stress state} \\ Y > 1 & \text{for } \varphi_{mob} \text{ higher than the critical stress state} \end{array} \tag{7}$$

Therefore, $w_y = 1$ at the critical stress state and for higher mobilised friction angles ($\varphi_{mob} \geq \varphi_c$), $w_y = 0$ at the isotropic stress state and $0 < w_y < 1$ for $Y_i < Y < 1$. The exponential interpolation is controlled by a new model parameter ξ. Eq. (6) is graphically represented in Fig. 1.

$\vec{\mathbf{D}}_{SOM}$ in (5) is the direction of stretching at swept-out-memory conditions that correspond to the current stress state $\hat{\mathbf{T}}$, calculated using a procedure developed in [9]:

$$\vec{\mathbf{D}}_{SOM} = -\frac{\boldsymbol{\mathcal{A}}^{-1} : \mathbf{N}}{\|\boldsymbol{\mathcal{A}}^{-1} : \mathbf{N}\|} \tag{8}$$

where the fourth-order tensor $\boldsymbol{\mathcal{A}}$ is defined as

$$\boldsymbol{\mathcal{A}} = fs\boldsymbol{\mathcal{L}} + \frac{1}{\lambda^*}\mathbf{T} \otimes \mathbf{1} \tag{9}$$

with λ^* being a parameter of the original model. The proposed formulation has the following properties:

1. At the isotropic stress state $Y = Y_i$, therefore $w_y = 0$ and because $\vec{\mathbf{D}}_{SOM} = \vec{\mathbf{1}}$, $D_n = |\vec{\mathbf{1}} : \mathbf{D}|$. For the isotropic loading and unloading $\vec{\mathbf{D}} = \mathbf{D}/\|\mathbf{D}\| = \pm\vec{\mathbf{1}}$, therefore $D_n = \|\mathbf{D}\|$ and predictions by the original model are recovered. For undrained loading ($\operatorname{tr}\mathbf{D} = 0$), however, $D_n = 0$, so predictions are controlled by the linear part $fs\boldsymbol{\mathcal{L}} : \mathbf{D}$ only and since the tensor $\boldsymbol{\mathcal{L}}$ is isotropic, $\dot{p} = 0$ is predicted. In other words, hypoelastic response envelope defined by $f_s\boldsymbol{\mathcal{L}}$ is squeezed for $\operatorname{tr}\mathbf{D} < 0$, elongated for $\operatorname{tr}\mathbf{D} > 0$ and unchanged for $\operatorname{tr}\mathbf{D} = 0$. For equivalent parameters it coincides with the response envelope of the model by Huang et. al. The response envelopes of the original and modified models for normally consolidated soil at the isotropic stress state are shown in Fig. 2. The parameters for London Clay from [7] (Tab. 1) are used throughout this paper (Figs. 2-7), with $\xi = 0.6$ when demonstrating predictions by the modified model.

2. At the critical stress state and higher φ_{mob} we get $Y \geq 1$, so $w_y = 1$ and $D_n = \|\mathbf{D}\|$ for any direction of stretching. In this case the predictions by the original and modified model coincide.

3. At swept-out-memory conditions (defined in the stress vs. void ratio space) $\vec{\mathbf{D}} = \vec{\mathbf{D}}_{SOM}$, therefore $|\vec{\mathbf{D}}_{SOM} : \mathbf{D}| = \|\mathbf{D}\|$ and $D_n = \|\mathbf{D}\|$ for any value of w_y. The predictions by the original and modified models coincide.

4. For other cases the terms $\|\mathbf{D}\|$ and $|\vec{\mathbf{D}}_{SOM} : \mathbf{D}|$ in (5) are weighted by w_y, the relative importance of both terms is controlled by the new parameter ξ.

Thanks to the condition 3 the modified model predicts the same swept-out-memory surface [9] (and, therefore, normal compression lines in the $\mathbf{T}$ vs. e space) as the original model. The parameter ξ controls the shape of the undrained stress path. Its influence on undrained stress path for a normally consolidated clay starting from an isotropic stress state is demonstrated in Fig. 3.

2.1 Inspection into the model properties

Eqs. (4) and (5) may be re-written in the form similar to Eq. (2)

$$\overset{\circ}{\mathbf{T}} = f_s \left(\boldsymbol{\mathcal{L}}^D : \mathbf{D} + w_y f_d \mathbf{N} \|\mathbf{D}\| \right) \tag{10}$$

where the hypoelastic tensor $\boldsymbol{\mathcal{L}}^D$ is now dependent on the direction of stretching (with respect to $\vec{\mathbf{D}}_{SOM}$) via a bi-linear equation

$$\boldsymbol{\mathcal{L}}^D = \begin{cases} \boldsymbol{\mathcal{L}} + f_d \left(1 - w_y\right) \left(\mathbf{N} \otimes \vec{\mathbf{D}}_{SOM}\right) & \text{for} \quad \vec{\mathbf{D}}_{SOM} : \mathbf{D} > 0 \\ \boldsymbol{\mathcal{L}} - f_d \left(1 - w_y\right) \left(\mathbf{N} \otimes \vec{\mathbf{D}}_{SOM}\right) & \text{for} \quad \vec{\mathbf{D}}_{SOM} : \mathbf{D} \leq 0 \end{cases} \tag{11}$$

Therefore, the response envelope by the modified model is similarly to the envelope by the model by Huang et al. composed of two elliptic sections. At the isotropic stress state $w_y = 0$ and the two elliptic sections are centered about the reference stress state. For higher stress ratios, with $0 < w_y < 1$, the non-linear part is activated and the centres of the two elliptic sections are translated with respect to the reference stress state. The response envelope of the modified model at anisotropic stress state is plotted in Fig. 4. The envelope corresponds to the K_0 normally compressed state, the two elliptic sections are in Fig. 4 depicted by thin dotted lines.

Eq. (11) reveals that the difference between the two elliptic sections (and, therefore, difference in predictions by the original and modified models) decreases with increasing w_y (increasing φ_{mob}) and with decreasing value of the *pyknotropy* factor f_d. In the original model the factor f_d is directly related to the overconsolidation ratio $OCR = p_e^*/p$, where p_e^* is the Hvorslev equivalent pressure on the isotropic normal compression line

$$f_d = \left(\frac{2}{OCR} \right)^\alpha \tag{12}$$

Therefore, the predictions by the original and modified models do not differ substantially for higher $OCRs$, see response envelopes plotted for isotropic stress states at $OCR = 2$ and 4 in Fig. 5.

As the difference between the original and modified models fades away with increasing stress deviator and increasing OCR, the predictions of the two models are not significantly different at K_0 initial conditions. At K_0 normally compressed state the higher stress deviator suppresses the influence of the modification, at K_0 overconsolidated states the two models yield similar predictions due to higher $OCRs$. This is demonstrated in Fig. 6, where the response envelopes are plotted along K_0 unloading line.

The influence of the OCR on the shape of undrained stress paths predicted by the original and modified models for compression and extension tests starting from an isotropic stress state is shown in Fig. 7. The shapes of the stress paths on normally consolidated and slightly overconsolidated soils are more realistic, while the model still correctly captures the qualitative influence of overconsolidation.

3 Evaluation of the model

3.1 Isotropic stress state

In this section the model will be evaluated with respect to experimental data on normally consolidated Fujinomori Clay by Nakai et al. [11]. For details on calibration of the original hypoplastic model and the model by Huang et al. using drained compression test the reader is referred to Reference [5]. The parameters of the original model are given in Tab. 1. It may be seen from Fig. 8 that the parameters of the original model evaluated by Huang et al. are suitable for predicting the drained test also with the modified model by assuming the new parameter $\xi = 0.6$. In fact, the predictions by the original and modified models along drained stress path are very similar and reproduce well the observed soil behaviour. The only discrepancy is that the compressive volumetric strains are underpredicted by the original and modified models. This is due to a fixed position of the isotropic normal compression line with respect to the critical state line in the $\ln p$ vs. $\ln(1+e)$ space. If desired, a new parameter that would allow us to controll its position could easily be introduced into the models by modifying the pyknotropy factor f_d (Eq. (12)). The new parameter would have similar effect as parameter n of the model by Huang et al.

The parameters from Tab. 1 were used for simulation of undrained compression and extension tests on Fujinomori Clay (Fig. 9). It is clear that the predictions of the modified model for tests on normally consolidated soils starting from the *isotropic* stress states are significantly different as compared to the original model. With parameters evaluated using drained tests the modified model captures well the soil behaviour under undrained stress paths, and its predictions are very similar to the model by Huang et al.

3.2 Anisotropic stress state

Figure 10 shows the experimental results of undrained compression and extension experiments on reconstituted Bothkennar Clay by Allman and Atkinson [1]. The specimens were under K_0 conditions normally consolidated and then unloaded to four different $OCRs$. The simulations by the original and modified models with parameters evaluated in [8] with slightly increased value of the parameter κ^* (Tab. 1) and $\xi = 0.6$ are shown in Figs. 10 (a) and (b). The figures also include the swept-out-memory surfaces predicted by the original and modified models. It is clear that at K_0 initial conditions both models yield similar results, as discussed in Sec. 2.1. The shape of the undrained stress paths may be considered as satisfactory, at least from the qualitative point of view. For comparison, Fig. 10 (c) shows predictions by the model by Huang et al. The predictions are similar with predictions by the original and modified models for undrained compression from the K_0 normally compressed state only. Fig. 10 (c) clearly demonstrates two important shortcomings of the Huang et al. model – the positions of the critical state lines are significantly different for compression and for extension and the influence of OCR is not taken into account.

A shortcoming of the original and modified models is demonstrated in Fig. 11, which shows the stress-strain curves of the tests analysed (only predictions by the original model are shown, the predictions by the modified model are not significantly different). The stiffness is predicted satisfactorily for compression experiments, but the models significantly underestimate the decrease of the shear stiffness in undrained extension. As discussed by Niemunis [12], improvement of predictions of undrained tests from K_0 states would require more radical changes of the model, namely the incorporation of anisotropy. This is outside the scope of this paper.

4 Concluding remarks

This contribution presents a conceptually simple modification of the hypoplastic model for clays by Mašín [7]. The modification is aimed to improve performance of the model at low

OCRs, particularly to improve predictions under undrained conditions for isotropically consolidated specimens. At normally consolidated states, the model yields similar results as a recently proposed hypoplastic model by Huang et al. [5], however, the modified model can also be used to predict the behaviour of overconsolidated soil. Also, the proposed modification has no negative impact on the predictions of the swept-out-memory states by the original model.

It should be pointed out that the modified model does not change significantly predictions by the original model for specimens with K_0 initial states. As the K_0 states are present in the ground, the proposed modification cannot be expected to have significant impact on practical application of the hypoplastic model by Mašín [7]. Rather, the modification can be regarded as an inspection into some properties of hypoplastic models in general.

5 Acknowledgment

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Table 1: *Parameters of the original hypoplastic model used in this paper (references on the original calibration of the model).*

	φ_c	λ^*	κ^*	N	r
London Clay [7]	22.6°	0.11	0.016	1.375	0.4
Fujinomori Clay [5]	34°	0.0445	0.0108	0.8867	1.3
Bothkennar Clay [8]	35°	0.119	0.006	1.344	0.07

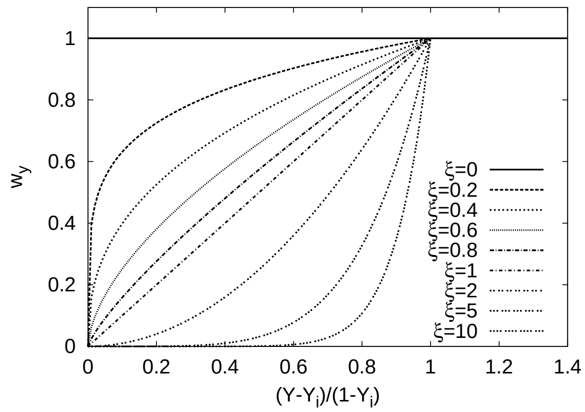


Figure 1: Influence of the new parameter ξ on weighting factor w_y (6).

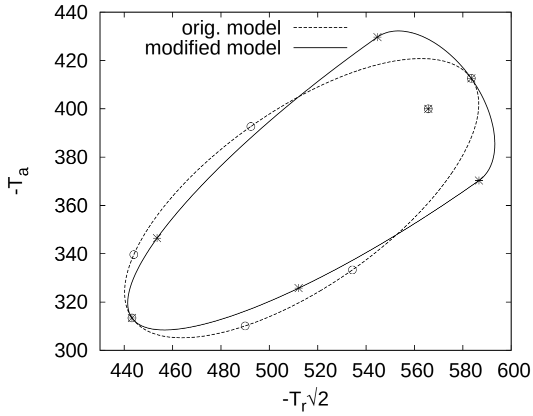


Figure 2: Response envelopes of normally consolidated soil at the isotropic stress state predicted by the original and modified models. The symbols in the above figure and in Figs. 4-6 stand for isotropic loading and unloading, undrained compression and extension and states with $D_a = 0$ and $D_r = 0$ for $\operatorname{tr}\mathbf{D} > 0$.

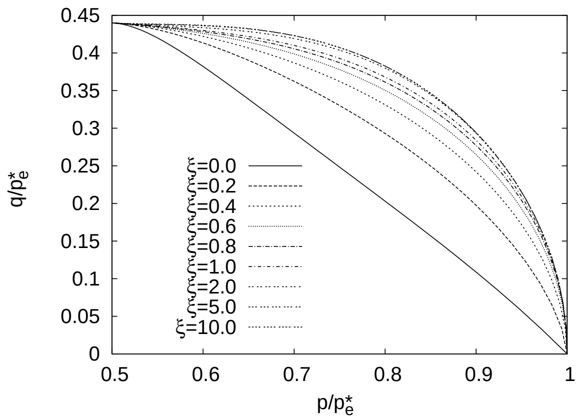


Figure 3: The influence of the parameter ξ on the shape of the undrained stress path of normally consolidated soil starting from the isotropic stress state. p_e^* is the Hvorslev equivalent pressure on the isotropic normal compression line.

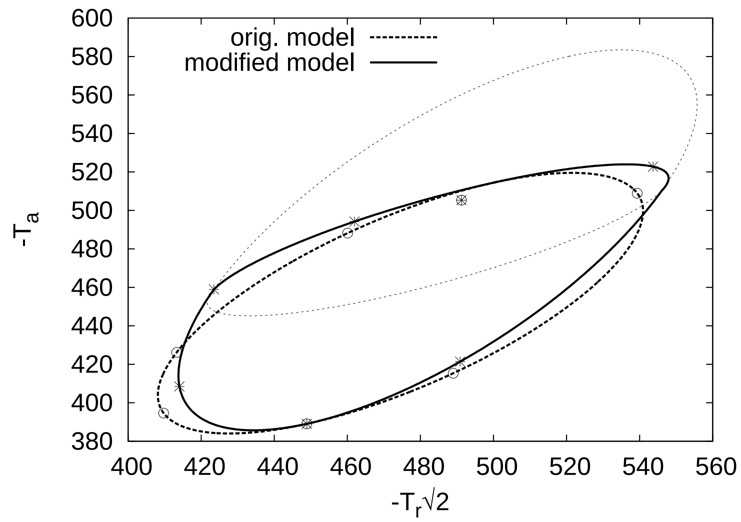
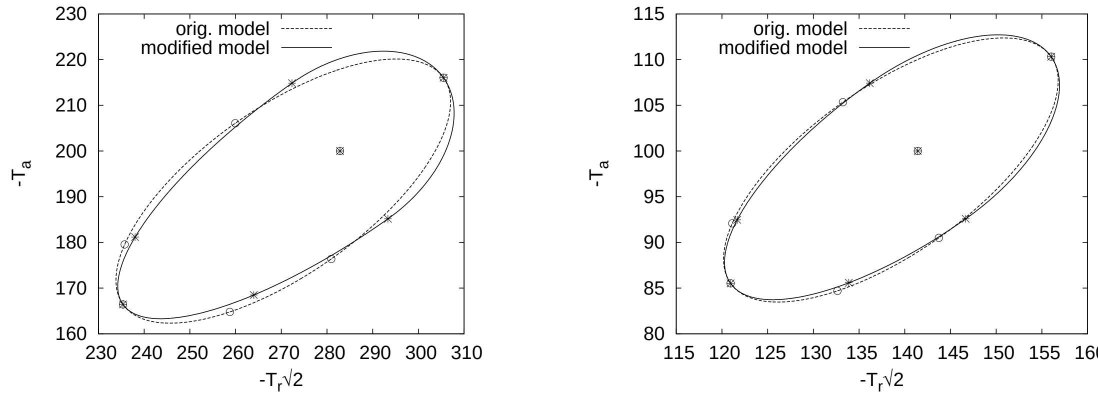


Figure 4: Response envelopes of normally consolidated soil at the K_0 stress state predicted by the original and modified models.



(a) (b)

Figure 5: Response envelopes at the isotropic stress state predicted by the original and modified models for $OCR = 2$ (a) and 4 (b). For $OCR = 1$ see Fig. 2.

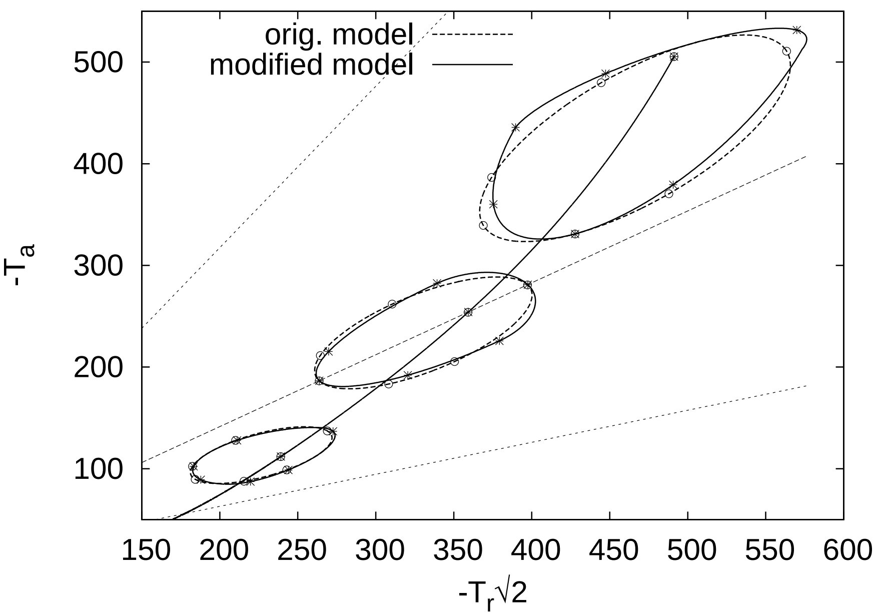


Figure 6: Response envelopes at K_0 states with different OCR predicted by the original and modified models. K_0 path by the original model.

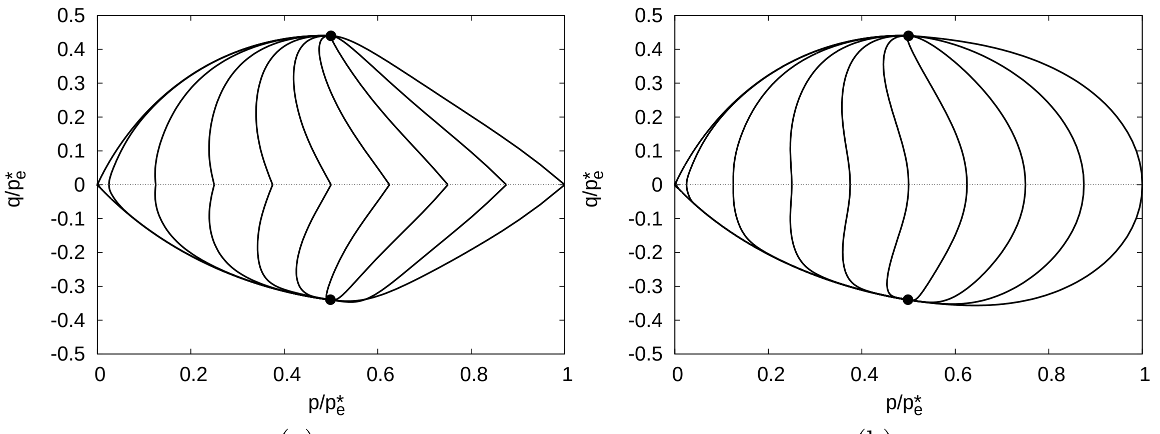


Figure 7: Stress paths of undrained compression and extension tests with different initial values of $OCRs$. The original model (a) and the modified model with $\xi = 0.6$ (b).

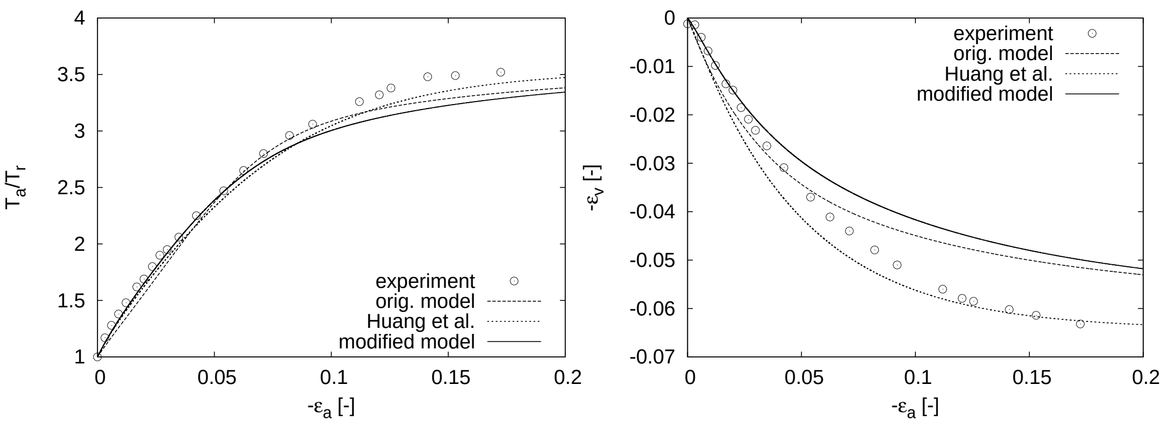


Figure 8: Drained compression test from the isotropic normally consolidated state on Fujinomori Clay [11], predictions by the original, modified and Huang et al. models

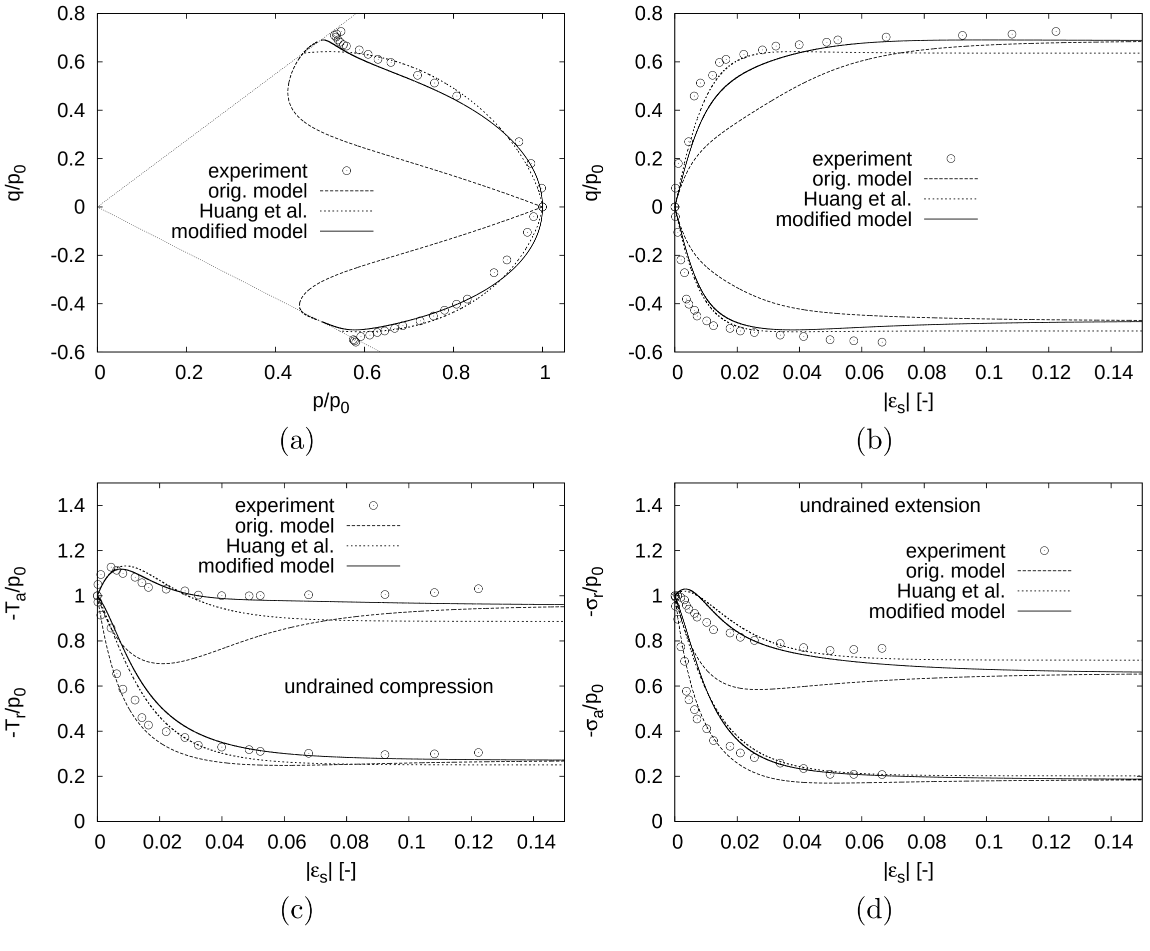
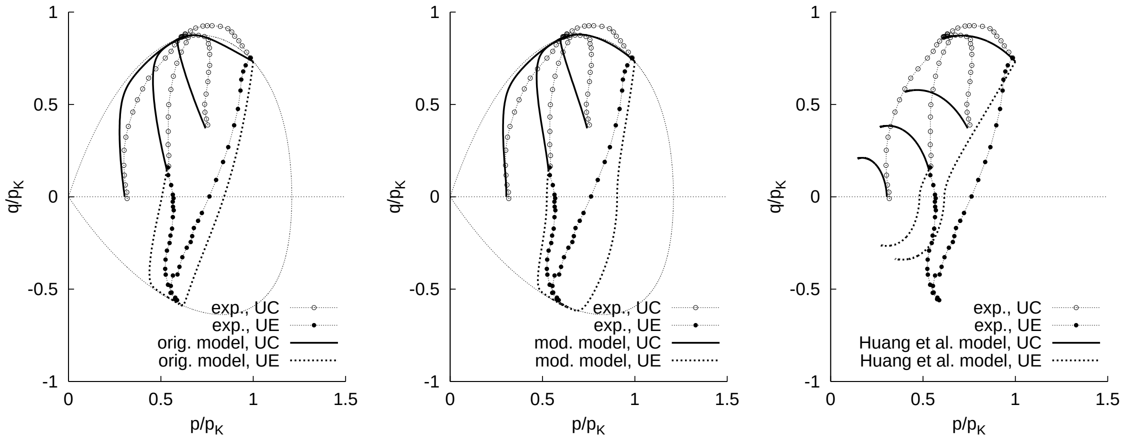
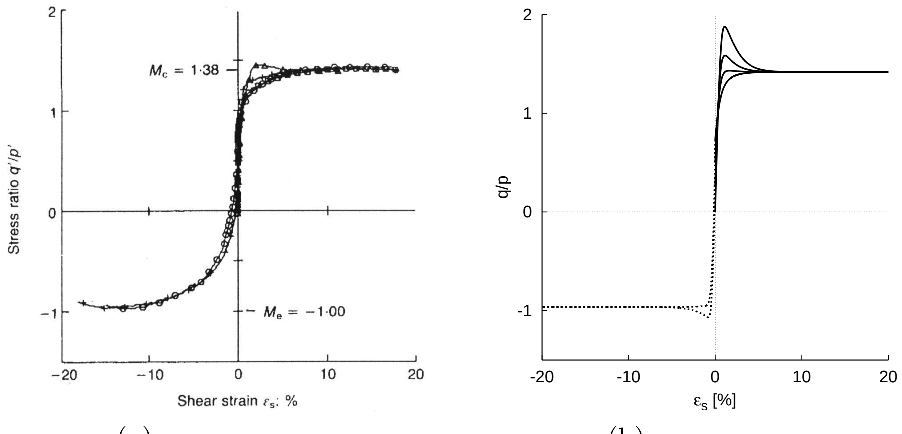


Figure 9: Undrained compression and extension tests from the isotropic normally consolidated state on Fujinomori Clay [11], predictions by the original and modified models and model by Huang et al. p_0 is the initial mean stress.



(a) (b) (c)

Figure 10: Undrained compression and extension experiments from K_0 states on Bothkennar Clay [1], stress paths normalised with respect to equivalent pressure on the K_0 normal compression line. Predictions by the original (a), modified (b) and Huang et al. (c) models. p_K is the equivalent mean stress at the K_0 normal compression line.



(a) (b)

Figure 11: Undrained compression and extension experiments from K_0 states on Bothkennar Clay [1] (a), ϵ_s vs. q/p diagrams. Predictions by the original model (b).