Modelling of thermal effects in hypoplasticity

D. Mašín
Charles University, Prague, Czech Republic

N. Khalili
University of New South Wales, Sydney, Australia

Abstract

The paper presents a mechanical model for non-isothermal behaviour of variably saturated soils. The model is based on an incrementally non-linear hypoplastic model for saturated clays and can therefore tackle the non-linear behaviour of overconsolidated soils. A hypoplastic model for non-isothermal behaviour of saturated soils was developed and combined with the existing hypoplastic model for unsaturated soils based on the effective stress principle. The number of model parameters is kept to a minimum, and they all have a clear physical interpretation, to facilitate the model usefulness for practical applications. The step-by-step procedure used for the parameter calibration is described. The model is finally evaluated using a comprehensive set of experimental data for the thermo-mechanical behaviour of a variably saturated compacted silt.

1 INTRODUCTION

Understanding and modelling of thermo-mechanical properties of soils, particularly fine grained materials, has been the subject of many studies in the past. The reason for this attention is the non-isothermal conditions encountered in a number of high-priority applications such as nuclear waste disposal storage, buried high-voltage cables, pavements, and geothermal energy. Majority of the constitutive models for variably saturated soils under non-isothermal conditions have been developed within elasto-plastic framework with reversible response inside the Yield surface. These models have well-known drawbacks, particularly in their inability to capture the highly non-linear behaviour of overconsolidated soils.

The aim of this paper is to demonstrate an approach to incorporating the thermal effects into an existing hypoplastic model for variably saturated soils (Mašín and Khalili 2008). The model predicts not only the qualitative effects of temperature and suction on the soil behaviour at large strains, but also correctly captures the non-linear soil behaviour in the medium- to small-strain range. The number of model parameters is kept to a minimum, to facilitate its usefulness for practical applications.

Comprehensive experimental data on the thermo-mechanical behaviour of unsaturated soils is scarce in the scientific literature. On the other hand, a number of researchers have studied thermo-mechanical soil behaviour under saturated conditions. The following characteristics of the thermal soil response appear to be the most important:

- Temperature influences the normal compression lines (NCL) of a soil. A majority of the experimental data show that in the applied stress range they may be considered parallel to each other, while with increasing temperature the specific volume at the NCL for the given effective mean stress p decreases.
- The experimental evidence of the influence of temperature on the soil peak strength is contradictory, but most results agree that the critical state friction angle is independent of temperature.
- The soil response to heating-cooling cycles is strongly dependent on the apparent overconsolidation ratio. At high OCRs, the soil response is essentially reversible, thus, there are no permanent changes in the soil structure. As discussed in detail by Khalili et al. (2010), this type of response is controlled solely by the thermal expansion coefficient of the solid particles, and it is independent of the soil porosity. An interesting consequence of this fact is that heating or cooling of overconsolidated soils imposes no change in the porosity (Khalili et al. 2010).
- At low OCRs, the mechanisms controlling the heating and cooling responses are substantially different. Upon cooling, the state boundary surface increases in size; the soil structure is thus not exposed to meta-stable conditions, and consequently the volumetric response is the result of the thermal contraction of the soil particles only. In contrast, a reduction of the size of SBS due to heating imposes irreversible changes of the open structure of a soil at low OCR, leading to the so-called heating-induced collapse. The collapse due to heating is not an abrupt process that activates once the soil state reaches SBS; instead, its influence gradually increases with decreasing OCR.

All the above effects are considered by the newly developed constitutive model.

2 REFERENCE HYPOPLASTIC MODEL FOR SATURATED SOILS

Hypoplasticity is a particular class of incrementally non-linear constitutive models for soils developed independently at the Universities of Karlsruhe and Grenoble (see Tamagnini et al. 2000). Unlike the elasto-plastic models, the strain rate is not decomposed into reversible (elastic) and irreversible (plastic) parts, and the incrementally non-linear character of the soil behaviour is reproduced by the general equation for the stress rate, which is non-linear in the strain rate $\dot{\boldsymbol{\epsilon}}$. The reference model for the present derivations was proposed by Mašín (2005) and it is based on the Karlsruhe approach to hypoplasticity. Within this context, the stress-strain rate relationship is written as

$$\mathring{\boldsymbol{\sigma}} = f_s(\mathcal{L} : \dot{\boldsymbol{\epsilon}} + f_d \mathbf{N} \|\dot{\boldsymbol{\epsilon}}\|) \tag{1}$$

where $\mathring{\boldsymbol{\sigma}}$ denotes the objective rate of the effective stress tensor, $\dot{\boldsymbol{\epsilon}}$ is the Euler's stretching tensor, $\mathcal{L}$ and $\mathbf{N}$ are fourth- and second-order constitutive tensors and f_s and f_d are two scalar factors, denoted as *barotropy* and *pyknotropy* factors respectively. A detailed description of these factors and the mathematical structure of the model is outside the scope of this paper and the readers are referred to the relevant publication (Mašín 2005).

The model is conceptually based on critical state soil mechanics (see Gudehus and Mašín 2009) and its five parameters (φ_c, N, λ^*, κ^*, r) have a similar physical interpretation as the parameters of the Modified Cam clay model. Parameters N and λ^* define the position and the slope of the isotropic normal compression line

$$\ln(1+e) = N - \lambda^* \ln \frac{p}{p_r} \tag{2}$$

where p_r is an arbitrary reference stress, which is considered equal to 1 kPa throughout this paper. The parameters N and λ^* also control the position of the critical state line, with the assumed formulation

$$\ln(1+e) = N - \lambda^* \ln \frac{p}{2p_r} \tag{3}$$

The next parameter, κ^*, controls the slope of the isotropic unloading line and the parameter r the shear stiffness. Finally, φ_c is the critical state friction angle that controls the size of the critical state locus in the stress space. The model considers the void ratio e as a state variable.

3 THERMOMECHANICAL MODEL FOR SATURATED SOILS

To account for the influence of temperature on the apparent preconsolidation and the size of the state boundary surface, the model parameters controlling the position and slope of the normal compression line N and λ^* (Eq. (2)) are considered to be dependent on temperature. The normal compression line thus has the following formulation:

$$\ln(1+e) = N(T) - \lambda^*(T) \ln \frac{p}{p_r} \tag{4}$$

The following relation is adopted in the present work:

$$N(T) = N + n_T \ln\left(\frac{T}{T_0}\right)$$

$$\lambda^*(T) = \lambda^* + l_T \ln\left(\frac{T}{T_0}\right) \tag{5}$$

in which T_0 is a reference temperature, and the values of $N(T_0)$ and $\lambda^*(T_0)$ corresponding to the reference temperature T_0 are model parameters, which are for brevity denoted as N and λ^*. n_T and l_T are additional parameters controlling the influence of temperature on NCL. It is noted that according to the experimental evidence, the slope of the NCL for most practical problems may be taken as independent of temperature (thus $l_T = 0$). This means that, in order to predict a decrease of the preconsolidation pressure with increasing temperature, n_T should be negative.

When the overconsolidated saturated soil, which is not prone to heating-induced collapse of the soil structure, is heated under drained conditions, it undergoes thermal expansion. Khalili et al. (2010) have shown that the overall thermal expansion coefficient of a porous medium α_s is solely controlled by, and is equal to, the thermal expansion coefficient of the solid constituent. It follows that the thermal expansion is independent of the void ratio, has no effect on the void ratio, and is fully reversible. The available experimental data also shows that the coefficient α_s may essentially be considered as independent of the effective stress and temperature. We assume a thermally isotropic material, thus:

$$\dot{\boldsymbol{\epsilon}}^{TE} = \frac{\mathbf{1}}{3} \alpha_S \dot{T} \tag{6}$$

To account for the full reversibility of the $\dot{\boldsymbol{\epsilon}}^{TE}$ strain rate, the hypoplastic formulation (1) is modified in the following way:

$$\mathring{\boldsymbol{\sigma}} = f_s\left[\mathcal{L} : (\dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}^{TE}) + f_d \mathbf{N} \|\dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}^{TE}\|\right] \tag{7}$$

The reversible component of the thermal strain rate does not imply any change in the void ratio of the porous medium. To account for this phenomenon, the rate of the void ratio is not calculated from the total strain rate $\dot{\boldsymbol{\epsilon}}$, but it is given by

$$\dot{e} = (1+e)\operatorname{tr}(\dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}^{TE}) \tag{8}$$

In addition to the reversible strains induced by the volumetric change of the solid constituents, a soil with a high void ratio (low overconsolidation ratio) is prone to heating-induced irreversible compression of the soil structure. In terms of critical state soil mechanics, this is manifested by the reduction of the size of the state boundary surface. As demonstrated by Mašín and Khalili (2008), collapse of the soil structure at constant effective stress may be incorporated into hypoplasticity through an additional tensorial term $\mathbf{H}$. For the states at the state boundary surface, the collapsible term due to heating $\mathbf{H}_T$ is incorporated by

$$\overset{\circ}{\boldsymbol{\sigma}} = f_s\left[\mathcal{L} : (\dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}^{TE}) + f_d \mathbf{N}\left\|\dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}^{TE}\right\|\right] + \mathbf{H}_T \tag{9}$$

The calculation of $\mathbf{H}_T$ follows from the requirement that when the normally consolidated soil is heated under constant effective stress, its state must remain on the state boundary surface. In other words, stress normalised by the size of the state boundary surface ($\boldsymbol{\sigma}_n$) must not change. The size of the state boundary surface is measured by the Hvorslev equivalent pressure p_e on the normal compression line, which follows from (4):

$$p_e = p_r \exp\left[\frac{N(T) - \ln(1+e)}{\lambda^*(T)}\right] \tag{10}$$

The rate of the normalised stress state $\boldsymbol{\sigma}_n = \boldsymbol{\sigma}/p_e$ is given by

$$\overset{\circ}{\boldsymbol{\sigma}}_n = \frac{\overset{\circ}{\boldsymbol{\sigma}}}{p_e} - \frac{\boldsymbol{\sigma}}{p_e^2}\dot{p}_e \tag{11}$$

and the rate of the Hvorslev equivalent pressure $\dot{p}_e$ follows from (10):

$$\dot{p}_e = -\frac{p_e}{\lambda^*(T)}\operatorname{tr}\dot{\boldsymbol{\epsilon}} + \frac{\partial p_e}{\partial T}\dot{T} \tag{12}$$

As already indicated, heating of the soil whose state lies at the state boundary surface must impose no change in $\boldsymbol{\sigma}_n$. Combination of (11) with (12) thus yields

$$\overset{\circ}{\boldsymbol{\sigma}} = -\frac{\boldsymbol{\sigma}}{\lambda^*(T)}\operatorname{tr}\dot{\boldsymbol{\epsilon}} + \frac{\boldsymbol{\sigma}}{p_e}\frac{\partial p_e}{\partial T}\dot{T} \tag{13}$$

The basic hypoplastic model is characterised by $\partial p_e/\partial T = 0$. It thus follows that the relation

$$\overset{\circ}{\boldsymbol{\sigma}} = -\frac{\boldsymbol{\sigma}}{\lambda^*(T)}\operatorname{tr}\dot{\boldsymbol{\epsilon}} \tag{14}$$

gives the effective stress rate predicted by the basic hypoplastic model for compression paths along NCL. In general, this rate is given by Eq. (7). Since satisfaction of the condition $\overset{\circ}{\boldsymbol{\sigma}}_n = \mathbf{0}$ is inherent to the formulation of the reference model, comparison of (14), (13), (7) and (9) yields the expression for the term $\mathbf{H}_T$:

$$\mathbf{H}_T = \frac{\boldsymbol{\sigma}}{p_e}\frac{\partial p_e}{\partial T}\dot{T} \tag{15}$$

For the particular choice of the dependency of $N(T)$ and $\lambda^*(T)$ on temperature (5), we have

$$\mathbf{H}_T = \frac{\boldsymbol{\sigma}}{T\lambda^*(T)}\left[n_T - l_T \ln\frac{p_e}{p_r}\right]\dot{T} \tag{16}$$

Eqs. (16) and (9) define the thermo-mechanical hypoplastic model for normally consolidated conditions under constant temperature or heating ($\dot{T} \geq 0$). In order to generalise it for an arbitrary state and arbitrary loading conditions, it must be enhanced in the following way:

1. Collapse of the soil structure is an irreversible process that takes place during heating only. Upon cooling, volumetric contraction of the soil skeleton is controlled solely by the thermal contraction of the solid particles. Thus, the $\mathbf{H}_T$ term is active only for $\dot{T} \geq 0$, and Eq. (16) can be rewritten as

$$\mathbf{H}_T = \frac{\boldsymbol{\sigma}}{T\lambda^*(T)}\left[n_T - l_T \ln\frac{p_e}{p_r}\right]\langle\dot{T}\rangle \tag{17}$$

2. Collapse of the soil structure is most pronounced for a soil with an open structure, i.e. soil at low overconsolidation ratios. Thus, the influence of the $\mathbf{H}_T$ term should vanish with increasing OCR. To reflect this, Mašín and Khalili (2008) introduced a factor f_u which has the following property: $f_u = 1$ for $OCR = 1$ and $f_u \to 0$ for $OCR \to \infty$. The factor f_u multiplies the term $\mathbf{H}_T$ in the model formulation, and thus reduces its effect with increasing overconsolidation ratio:

$$\overset{\circ}{\boldsymbol{\sigma}} = f_s\left[\mathcal{L} : (\dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}^{TE}) + f_d \mathbf{N}\left\|\dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}^{TE}\right\|\right] + f_u\mathbf{H}_T \tag{18}$$

The following expression satisfies the outlined properties of the factor f_u:

$$f_u = \left(\frac{p}{p^{SBS}}\right)^m \tag{19}$$

where p^{SBS} is the effective mean stress at the SBS corresponding to the current normalised stress $\boldsymbol{\sigma}/\operatorname{tr}\boldsymbol{\sigma}$ and current void ratio e, and m is a model parameter controlling the influence of overconsolidation on the heating-induced collapse. The expression for f_u may be derived from the formulation of the pyknotropy factor f_d of the basic hypoplastic model:

$$f_d = \left(\frac{2p}{p_e}\right)^\alpha \tag{20}$$

Combining of (20) with (19) leads to

$$\frac{p}{p^{SBS}} = \left(\frac{f_d}{f_d^{SBS}}\right)^{1/\alpha} \tag{21}$$

where f_d^{SBS} is the value of the pyknotropy factor f_d at the state boundary surface corresponding to the current state. An analytical expression for f_d^{SBS} has been derived in Mašín and Herle (2005)

$$f_d^{SBS} = \left\| f_s \mathcal{A}^{-1} : \mathbf{N} \right\|^{-1} \tag{22}$$

where the fourth-order tensor $\mathcal{A}$ is given by

$$\mathcal{A} = f_s \mathcal{L} - \frac{1}{\lambda^*(T)} \boldsymbol{\sigma} \otimes \mathbf{1} \tag{23}$$

Therefore, the expression for the pyknotropy factor f_u reads

$$f_u = \left[f_d \left\| f_s \mathcal{A}^{-1} : \mathbf{N} \right\| \right]^{m/\alpha} \tag{24}$$

The influence of the parameter m on the value of the pyknotropy factor f_u is clear from Figure 1. For high values of m, collapse takes place very close to the state boundary surface only, and inside the SBS the thermally-induced strains are fully reversible. With decreasing value of m, the collapse takes place at progressively higher overconsolidation ratios.

3. The factor $\mathbf{H}_T$ defined by Eq. (17) satisfies the requirement of consistency of the model predictions at the SBS. However, the amount of collapse predicted by the model from Eq. (18) decreases with increasing OCR even for $m = 0$ (and thus $f_u = 1$). This property was found to be undesirable, since with (17) it is not possible to fully control the collapsible strains by the parameter m. To overcome this shortcoming, the factor $\mathbf{H}_T$ is modified for higher OCRs such that the amount of collapse for $m = 0$ (and thus $f_u = 1$) is independent of the overconsolidation ratio.

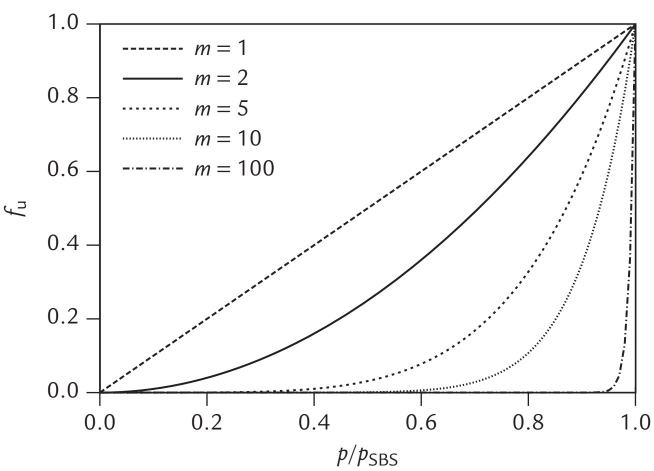

Figure 1 The influence of the parameter m on the value of the pyknotropy factor f_u (from Mašín and Khalili 2007).

For $f_u = 1$, $\mathring{\boldsymbol{\sigma}} = \mathbf{0}$ and $\dot{T} >= 0$ the formulation of the model (18) reads

$$-\mathbf{H}_T = f_s \left[\mathcal{L} : (\dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}^{TE}) + f_d \mathbf{N} \left\| \dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}^{TE} \right\| \right] \tag{25}$$

We now wish to modify the left-hand side of Eq. (25) in such a way that the volumetric response due to heating is independent of OCR. To achieve this, we multiply the left-hand side of Eq. (25) by a yet-unknown factor c_{ic}. We thus have

$$-c_{ic} \mathbf{H}_T = f_s \left[\mathcal{L} : (\dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}^{TE}) + f_d \mathbf{N} \left\| \dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}^{TE} \right\| \right] \tag{26}$$

We use the operator $-\mathrm{tr}(x)/3$ on both sides of Eq. (26); for details on manipulation with the right-hand side of Eq. (26) see Mašín (2005). Definition of the components $\mathcal{L}$, $\mathbf{N}$, f_s and f_d of the basic hypoplastic model is detailed in Mašín (2005). We finally obtain

$$c_{ic} \frac{\mathrm{tr}(\mathbf{H}_T)}{3} = \frac{p}{\lambda^*(T)} \left[\frac{3 + a^2 - f_d a \sqrt{3}}{3 + a^2 - 2^\alpha \sqrt{3}} \right] \frac{\dot{e}}{1+e} \tag{27}$$

Scalars a and α are defined in the Mašín (2005).

For the state at the state boundary surface, the following modifications of Eq. (27) apply: First, $c_{ic} = 1$. This is because in Eq. (25), upon which our derivations are based, the c_{ic} factor is equal to unity and the equation is valid for normally consolidated states. Second, for states at the SBS $f_d = f_d^{SBS}$ (f_d^{SBS} was defined in Eq. (22)). For states at the SBS we thus have

$$\frac{\mathrm{tr}(\mathbf{H}_T)}{3} = \frac{p}{\lambda^*(T)} \left[\frac{3 + a^2 - f_d^{SBS} a \sqrt{3}}{3 + a^2 - 2^\alpha \sqrt{3}} \right] \frac{\dot{e}}{1+e} \tag{28}$$

Comparing (28) with (27) leads us to

$$c_{ic} = \frac{3 + a^2 - f_d \sqrt{3}}{3 + a^2 - f_d^{SBS} \sqrt{3}} \tag{29}$$

The factor c_{ic} may be incorporated into the definition of the term $\mathbf{H}_T$, which now reads

$$\mathbf{H}_T = c_{ic} \frac{\boldsymbol{\sigma}}{T \lambda^*(T)} \left[n_T - l_T \ln \frac{p_e}{p_r} \right] \langle \dot{T} \rangle \tag{30}$$

The general rate formulation of the model is given by Eq. (18).

When compared with the reference hypoplastic model for constant temperature, the new model requires specification of additional parameters n_T and l_T (for temperature dependent NCL), α_s (thermal skeletal expansion coefficient), parameter m controlling the distance from the SBS upon which heating collapse takes place, and the reference temperature T_0. The model considers one additional state variable (temperature T). Evaluation of the material parameters is detailed in Sec. 1.

4 ADAPTATION OF THE MODEL TO UNSATURATED STATES

Experimental evidence shows that the temperature does not change the *qualitative* response of an unsaturated soil to a change in suction, and that suction does not change the *qualitative* response of the soil exposed to a change in temperature. Thanks to this property, constitutive models for the effects of unsaturation and temperature may be combined in a hierarchical way.

The proposed constitutive model for thermal effects on the soil behaviour has been combined with the existing hypoplastic model for variably saturated soils by Mašín and Khalili (2008). The model is based on the effective stress principle for unsaturated soils (Khalili and Khabbaz 1998). Its detailed description is outside the scope of this paper. The additional parameters describing the effects of variable saturation are as follows:

- Air-entry suction s_e.
- Parameters controlling the variation of the positions of normal compression lines with suction n_s and l_s.

5 DETERMINATION OF THE MODEL PARAMETERS

In this section, evaluation of the model parameters based on experimental data on variably saturated soils at different temperatures by Uchaipchat and Khalili (2009) is presented. Due to the space restrictions, calibration of the parameters describing the thermal effects (T_0, n_T, l_T, α_s and m) will be detailed only. The approach to calibrating of all the model parameters was as follows: First, the parameter s_e needed to calculate the effective stress in simulations of the other tests had to be selected. For the sake of simplicity, in the present evaluation it was considered as a material constant. The value $s_e = 18$ kPa, evaluated by Uchaipchat and Khalili (2009) from the position of water retention curve at $T = 25°C$, was considered as the most appropriate. Calibration of s_e was followed by calibration of the parameters controlling NCL and isotropic mechanical response of saturated soil (N, λ^*, κ^*) and positions and slopes of NCLs of unsaturated and heated soil (n_s, l_s, n_T and l_T). At this point it was possible to simulate the constant s and constant T shear experiments, which were used to calibrate the shear stiffness parameter r and the critical state friction angle φ_c. Finally, the thermal heating test of an overconsolidated soil was used to calibrate the skeletal thermal expansion coefficient α_s and heating- and wetting-induced collapse parameter m.

5.1 Parameters Controlling the Isotropic Mechanical Response at Constant T

As noted by Uchaipchat and Khalili (2009), the slopes of the normal compression lines for the tested silt were independent of suction and temperature for the stress range of interest. Figure 2 shows the results of several constant suction and constant temperature isotropic compression tests, plotted in the effective stress space. All the NCLs can be approximated by a linear representation in the $\ln p$ vs. $\ln(1 + e)$ plane with a unique slope of $\lambda^* = 0.06$. The independency of λ^* on T leads to $l_T = 0$.

Unlike the slope λ^*, the intercept N is clearly dependent on T. Its value for the reference temperature T_0 and $s = 0$ is $N = 0.772$. The dependency of $N(s, T)$ on the values of $\ln(T/T_0)$, used for evaluation of the parameter n_T, is shown in Figure 3 and leads to $n_T = -0.01$. The reference temperature is considered to be $T_0 = 25°C$.

5.2 Parameters Controlling the Heating-Cooling Response

In the model, the response due to changes of T is treated using different approaches depending on

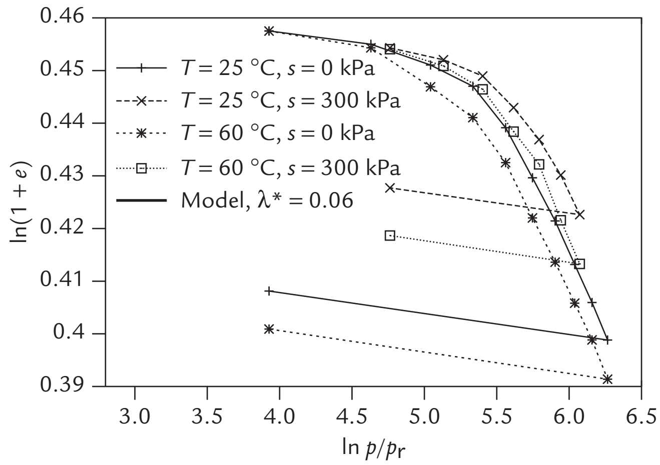

Figure 2 Results of the isotropic compression tests at different suctions and temperatures plotted in the effective stress space and model representation of NCLs.

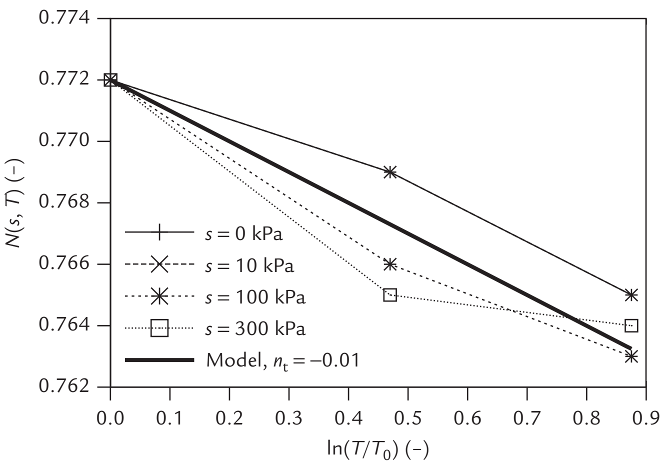

Figure 3 Calibration of parameter n_T.

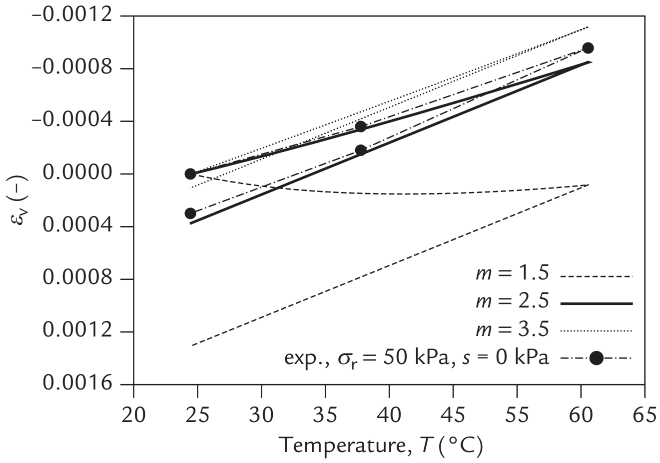

Figure 4 Calibration of the parameter m using heating-cooling experiments on an unsaturated soil at different stress levels (different overconsolidation ratios).

Table 1 Parameters of the proposed model for the silt investigated by Uchaipchat and Khalili (2009)

φ_c	λ^*	κ^*	N	r
29.5°	0.06	0.002	0.772	0.2
s_e	T_0			
18 kPa	25°C			
n_s	l_s	n_T	l_T	
0.0035	0	−0.01	0	
α_s		m		
3.5×10^{-5} °C^{-1}		2.5		

the actual value of OCR. The response due to heating-cooling cycles in an overconsolidated soil is fully reversible, controlled by the skeletal thermal expansion coefficient α_s. This was calibrated from the results of the heating-cooling experiment on an overconsolidated soil. The coefficient α_s was found to be independent of temperature and suction. The experimental data lead to $\alpha_s = 3.5 \times 10^{-5}$ (see slope of the cooling branch in Fig. 4).

The parameter m influences the distance from the SBS at which the collapsible behaviour due to heating takes place. It was calibrated using simulations of heating-cooling experiments on an overconsolidated soil (Fig. 4). The value of $m = 2.5$ appeared to best represent the observed behaviour.

The final set of all the material parameters is given in Table 1.

6 EVALUATION OF THE MODEL PREDICTIONS

All the simulations presented in the subsequent sections were performed using a single set of material parameters given in Table 1.

6.1 Temperature- and Suction-controlled Isotropic Loading Tests

Uchaipchat and Khalili (2009) performed a total of 12 isotropic compression tests at different temperatures

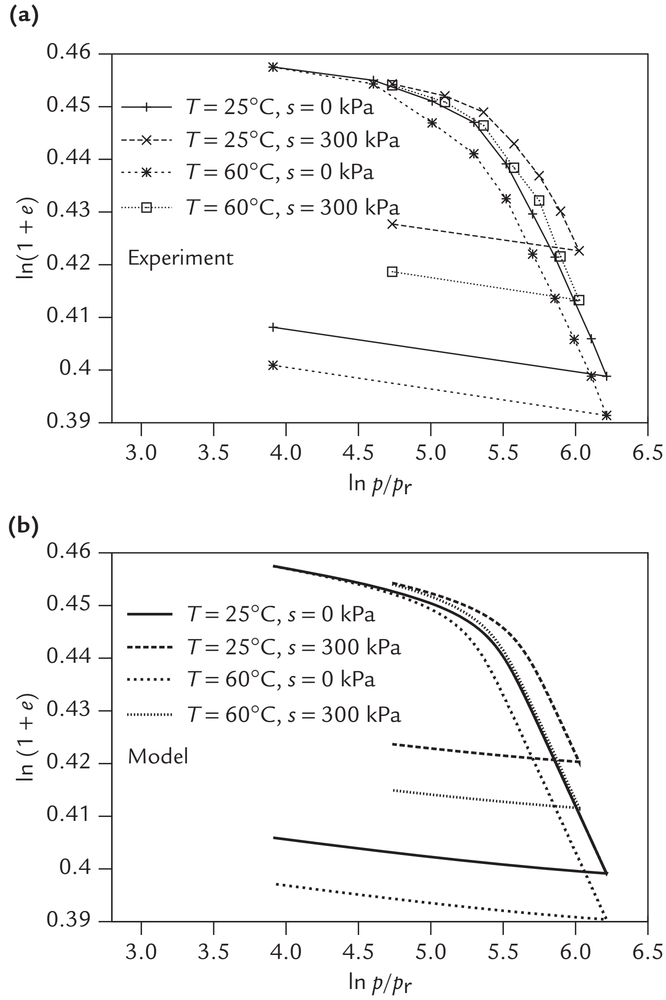

Figure 5 Comparison of experimental data (a) and simulations (b) of representative isotropic compression experiments at different suctions and temperatures.

(25°C, 40°C and 60°C) and different suctions (0 kPa, 10 kPa, 100 kPa and 300 kPa). Figure 5 shows the model predictions for four representative experiments (temperatures 25°C and 60°C and suctions 0 kPa and 300 kPa), with good agreement between the experimental data and model predictions.

6.2 Suction-controlled Thermal Loading and Unloading Tests

Volumetric behaviour due to heating and cooling was studied in thermal loading tests at constant suction. Figure 6 shows the results for the experiment on a saturated soil. The model correctly captures the increasing collapse potential with increasing overconsolidation ratio. The slope of the cooling branch is independent of both suction and overconsolidation ratio and it is controlled solely by the parameter α_s.

6.3 Temperature- and Suction-controlled Shear Tests

Figure 7 shows the results of drained triaxial shear experiments performed at different suction levels

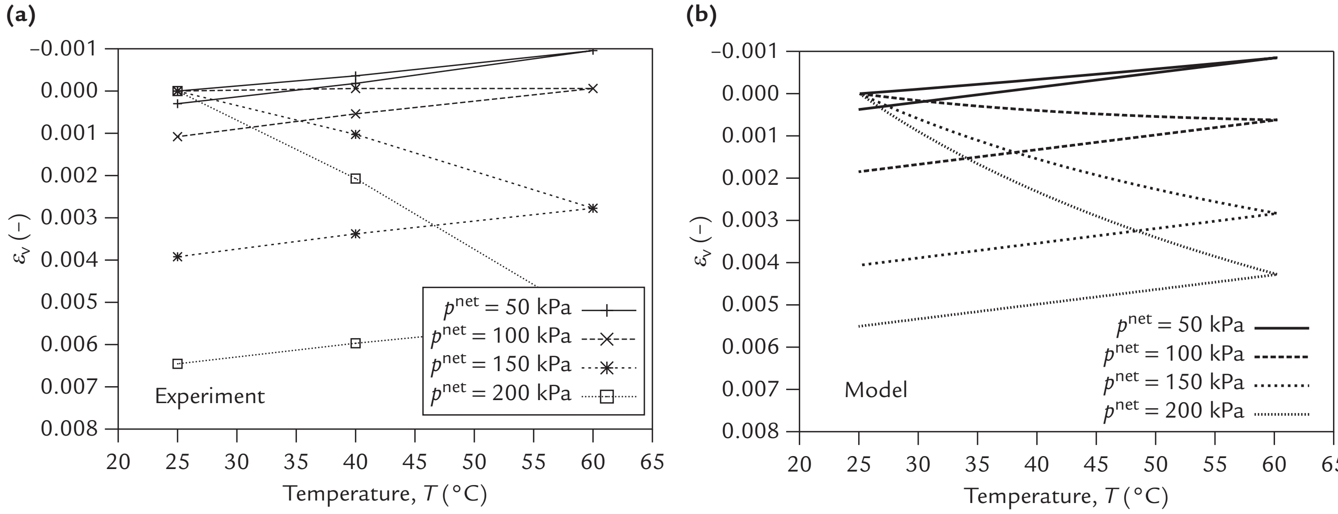

Figure 6 Volumetric change due to heating-cooling cycle in experiments at $s = 0$ kPa and different stress levels. (a) experiment, (b) model.

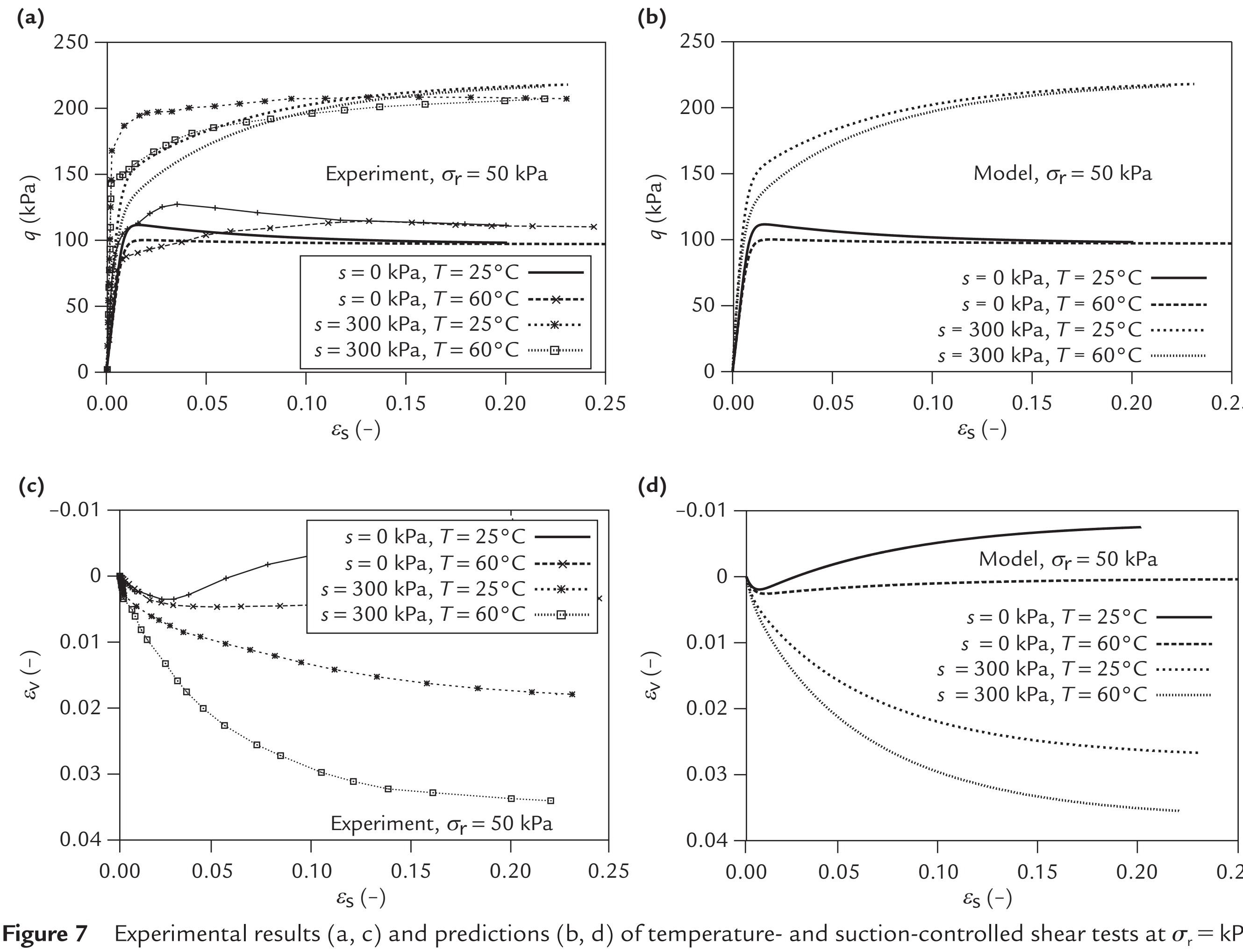

Figure 7 Experimental results (a, c) and predictions (b, d) of temperature- and suction-controlled shear tests at $\sigma_r =$ kPa.

and temperatures. The experimental data demonstrate that increasing temperature reduces the peak strength and induces more contractant response. This aspect is captured by the model, with good quantitative agreement between the experimental data and the model predictions.

6.4 Constant-water-content Thermal Loading Tests

Finally, the model has been evaluated by means of thermal loading tests under constant water content. Uchaipchat and Khalili (2009) performed a set of constant water content experiments on saturated soil samples (undrained tests). At saturated conditions, the volume change due to heating is caused by the difference in the thermal expansion coefficients of the solid particles α_s and water α_w:

$$\mathrm{tr}\,\dot{\epsilon} = [\alpha_w n + \alpha_s(1-n)]\dot{T} \quad (31)$$

where n is porosity $n = e/(1+e)$. The development of pore pressures is then controlled solely by the constitutive model for the soil mechanical behaviour. The coefficient α_w depends on both temperature and pressure. An empirical expression by Baldi et al. (1988) was adopted in the present simulations:

$$\alpha_w = \alpha_0 + (\alpha_1 + \beta_1 T)\ln mu_w + (\alpha_2 + \beta_2 T)(\ln mu_w)^2 \quad (32)$$

where u_w is pore water pressure in kPa and constants are given by $\alpha_0 = 4.505 \times 10^{-4}\,°C^{-1}$, $\alpha_1 = 9.156 \times 10^{-5}\,°C^{-1}$, $\beta_1 = -1.2 \times 10^{-6}\,°C^{-2}$, $\alpha_2 = 6.381 \times 10^{-6}\,°C^{-1}$, $\beta_2 = -5.766 \times 10^{-8}\,°C^{-2}$ and $m = 1.5 \times 10^{-6}\ \mathrm{kPa}^{-1}$.

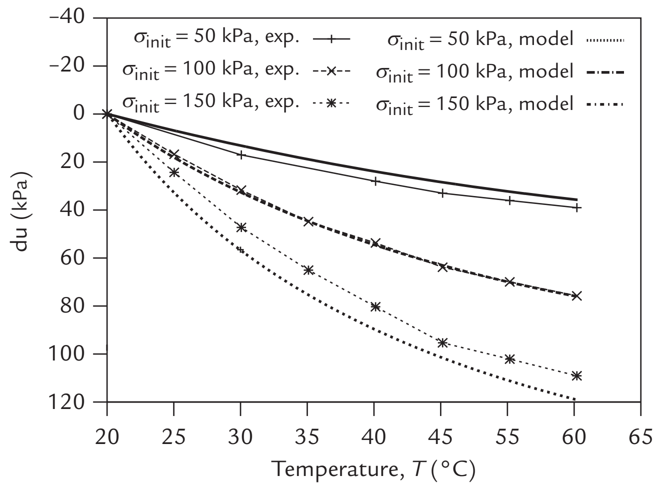

Figure 8 Experimental results and model predictions of pore pressure change during constant-water-content (undrained) heating experiments on saturated soil at different initial effective stresses (OCRs).

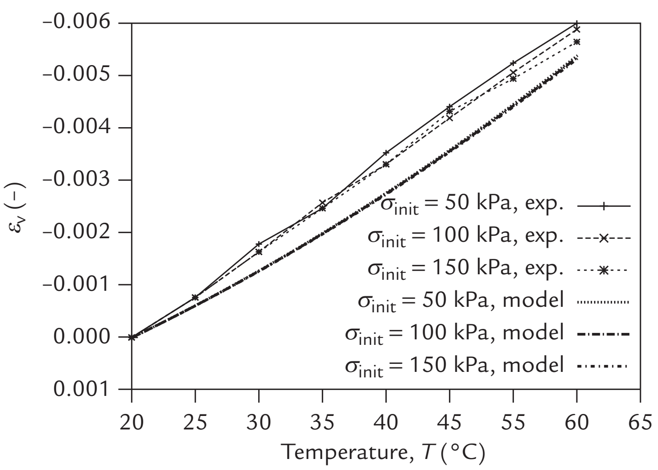

Figure 9 Experimental results and model predictions of void ratio and volumetric strain change during constant-water-content (undrained) heating experiments on saturated soil at different initial effective stresses (OCRs).

The experimental results and predictions of pore water pressures due to heating of saturated soils at different initial mean stresses are shown in Figure 8. The model prediction is in agreement with the experiment results, showing a higher decrease in the pore water pressure with an increase in the initial stress. This decrease is controlled by the soil bulk stiffness, which increases with effective stress.

The volumetric response is shown in Figure 9. Also the volumetric response is captured correctly by the model, which shows that the volumetric strain is practically independent of the applied effective stress.

7 CONCLUSIONS

In the paper, we presented a development of a constitutive model for the thermo-mechanical behaviour of variably saturated soils. Since the model is based on the incrementally non-linear hypoplastic model, it allows predictions of the non-linear soil behaviour in the medium- to large-strain range. Its extension to correctly predict the small- to very-small strain behaviour is straightforward (Niemunis and Herle 1997; Mašín 2005). The number of model parameters is kept to a minimum. All the parameters have a clear physical interpretation. The model was evaluated with respect to a comprehensive set of experimental data on variably saturated compacted silt given by (Uchaipchat and Khalili 2009). Not only can the model correctly predict of the non-linear response of overconsolidated soils, it can also deal with all the primary features of the mechanical behaviour of unsaturated soils under non-isothermal conditions.

ACKNOWLEDGMENT

The first author greatly appreciates a Visiting Fellow appointment in the School of Civil and Environmental Engineering of The University of New South Wales, Sydney, where this paper was originated. In addition, the financial support by the research grants of the Czech Science Foundation GACR P105/11/1884 and GACR 103/09/1262, and MSM 0021620855 is gratefully acknowledged.

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