Incorporation of meta-stable structure into hypoplasticity

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ABSTRACT: An existing approach to constitutive modelling of structured soils is in this paper applied to hypoplastic models. The proposed approach is based on the modification of the state boundary surface in such a way that the models predict different stress-dilatancy behaviour of structured and reference materials. The concept is applied to hypoplastic models for both clays and for granular materials. Good predictive capabilities of the modified model for clays are demonstrated, model for granular materials requires further modifications in order to take into account different small- to medium- strain behaviour of structured and reference materials.

1 INTRODUCTION

This paper introduces a conceptual framework for application of existing approaches to constitutive modelling of structured soils¹, developed usually within the framework of elasto-plasticity, to hypoplastic models (Kolymbas 1991). The paper discusses general aspects of the modelling of structural effects in hypoplasticity, its intention is neither compilation of the available experimental evidence regarding the behaviour of structured soils, nor thorough evaluation of predictive capabilities of the proposed constitutive models.

2 REFERENCE CONSTITUTIVE MODELS

The non-linear tensorial equation of the considered sub-class of hypoplastic models reads (Gudehus 1996)

$$\check{\mathbf{T}} = f_s \mathcal{L} : \mathbf{D} + f_s f_d \mathbf{N} \| \mathbf{D} \|$$
(1)

The models are formulated using two tensor-valued functions \mathcal{L} and \mathbf{N} and two scalar factors f_s and f_d . The soil state is characterised by the Cauchy stress (\mathbf{T}) and void ratio (e). The scalar functions f_s and f_d are named *barotropy* and *pyknotropy* factors (Gudehus 1996), which incorporate the influence of the mean stress and relative density (overconsolidation ratio), respectively.

The models characterised by Eq. (1) are suitable for predicting behaviour in the medium- to large-strain

range. In order to predict correctly the high stiffness in the small strain range, Eq. (1) must be modified, for example by the *intergranular strain concept* (Niemunis and Herle 1997). The application of the modified model is, however, outside the scope of the paper.

2.1 Hypoplastic model for clays

A hypoplastic constitutive model for clays (abbreviated HC) was developed by Mašín (2005a) as a modification of the hypoplastic model for soils with low friction angles by Herle and Kolymbas (2004).

The number of parameters of the model and their physical meaning corresponds to the parameters of the Modified Cam clay model, the non-linear character of the model, however, provides a significant improvement of predictions of the hypoplastic model with respect to the Modified Cam clay model, as demonstrated by Mašín et al. (2005).

The model makes use of the following five constitutive parameters: φ_c , N, λ^* , κ^* and r. φ_c is the critical state friction angle, the isotropic virgin compression line has the following formulation (Butterfield 1979):

$$\ln(1+e) = N - \lambda^* \ln\left(\frac{p}{p_r}\right) \tag{2}$$

with parameters N and λ^* and the reference stress $p_r = 1$ kPa. The parameters κ^* and r may be considered as factors that control bulk (κ^*) and shear (r) moduli of overconsolidated specimens.

2.2 Hypoplastic model for granular materials

A model by von Wolffersdorff (1996) (abbreviated VW) may be seen as a representative example of

¹In the following, the term 'structured soil' will be used for soil, which due to different fabric or additional bonding behaves differently compared to a 'reference' material. As the reference material equivalent soil reconstituted under standard conditions (Burland 1990) is usually considered.

hypoplastic models for granular materials developed at the University of Karlsruhe. Its calibration is described in detail in Herle and Gudehus (1999).

Model is in the stress-void ratio space characterised by three pressure-dependent limit states, describing the upper and lower bound of void ratio (e_i and e_d) and a critical state void ratio (e_c) (Fig. 1), defined using a power-law

$$\frac{e_i}{e_{i0}} = \frac{e_c}{e_{c0}} = \frac{e_d}{e_{d0}} = \exp\left[-\left(\frac{-\mathrm{tr}\mathbf{T}}{h_s}\right)^n\right] \qquad (3)$$

with parameters h_s , n, e_{i0} , e_{c0} and e_{d0} . In addition, three parameters are needed, namely critical state friction angle (φ_c) and parameters α and β , which control the influence of *pyknotropy*.



Figure 1. Pressure-dependent limit states of the VW model (Herle and Gudehus 1999)

3 CONCEPTUAL APPROACH FOR INCORPO-RATING META-STABLE STRUCTURE INTO CONSTITUTIVE MODELS

A conceptual framework for the behaviour of structured fine-grained soils was presented, e.g., by Cotecchia and Chandler (2000). Behaviour of structured granular materials, together with the interpretation by means of a constitutive model, was presented by Lagioia and Nova (1995).

Cotecchia and Chandler (2000) demonstrated that the influence of structure in fine-grained soils can be quantified by the different size of the state boundary surfaces² of the structured and reference materials (Fig. 2). Assuming a geometric similarity between the state boundary surfaces appears to be a reasonable approximation, although strongly anisotropic natural soils may exhibit SBS which is not symmetric about the isotropic axis. Similar findings were reported for granular materials by Lagioia and Nova (1995), who studied the behaviour of calcarenite. They observed that the state boundary surface of naturally cemented material has similar shape as the reconstituted soil, however SBS of the natural material is bigger and it



Figure 2. Theoretical framework for structured fine-grained materials (Cotecchia and Chandler 2000)

is shifted such that the state can access region of tensile stresses (Fig. 3).



Figure 3. Yield points and theoretical interpretation of natural and uncemented granular material (Lagioia and Nova 1995).

These observations are, in principle, applied in most of the currently available constitutive models for structured soils. In general, two additional state variables describing the effects of structure are needed, namely the ratio of sizes of SBSs of natural and reference materials, referred to as 'sensitivity' (s), and the shift of the SBS towards the region of tensile stresses measured along the isotropic axis, denoted by Lagioia and Nova (1995) p_t (with initial value p_{t0} , Fig. 3).

As s and p_t represent natural fabric and degree of bonding between soil particles, they may in the scale of engineering time only decrease or remain stationary. The limit values usually characterise the reference soil (s = 1 and $p_t = 0$), although higher values may be reasonable for soils with 'stable' elements of structure caused by natural fabric (Baudet and Stallebrass 2004). s and p_t are usually considered functions of accumulated plastic strain.

²State boundary surface (SBS) is defined as a boundary of all possible states of a soil element in the stress-void ratio space.

4 STATE BOUNDARY SURFACE IN HY-POPLASTICITY

State boundary surface is an important feature of soil behaviour that controls different stress-dilatancy responses of structured and reconstituted materials (Sec. 3). Elasto-plastic constitutive models incorporate state boundary surface, which has a pre-defined shape, explicitly by definition of a yield (or bounding in the case of advanced models) surface and a hardening law. On the other hand, hypoplastic models predict state boundary surface as a consequence of a particular choice of tensorial functions \mathcal{L} and \mathbf{N} and scalar factors f_s and f_d .

Mašín and Herle (2005b) studied the shape of the state boundary surface predicted by the hypoplastic model for clays. Using the concept of normalised incremental response envelopes they demonstrated that although the model predicts the state boundary surface, its explicit mathematical formulation is not available.

The model, however, allows us to derive mathematical formulation for swept-out-memory (limit) states, which may be considered as attractors of soil behaviour (Gudehus 1995). Limit states (defined by constant $\mathbf{\tilde{T}} = \mathbf{\tilde{T}}/||\mathbf{\tilde{T}}||$ and void ratio evolving along normal compression lines) are achieved asymptotically after sufficiently long proportional (constant $\mathbf{\tilde{D}}$) deformation paths. Mašín and Herle (2005b) demonstrated that in the stress-void ratio space limit states constitute a surface (named swept-out-memory (SOM) surface) and that this surface is a *close approximation of the state boundary surface*.

Because at swept-out-memory conditions $\mathbf{\tilde{T}} \parallel \mathbf{T}$, it is possible to introduce a scalar multiplier γ such that

$$\mathring{\mathbf{T}} = \gamma \vec{\mathbf{T}} \tag{4}$$

Eq. (1) therefore, at swept-out-memory conditions, reduces to

$$\gamma \vec{\mathbf{T}} = f_s \mathcal{L} : \mathbf{D} + f_s f_d \mathbf{N} \| \mathbf{D} \|$$
(5)

The second condition for SOM states (void ratio evolving along normal compression line) is in considered hypoplastic models described by

$$f_d = 0 \tag{6}$$

SOM conditions are for a particular hypoplastic model fully described if we solve Eqs. (4) and (5) such that for *given* **T** and $||\mathbf{D}||$ we find corresponding γ , $\vec{\mathbf{D}}$ and f_d . For HC and VW models solution was derived by Mašín and Herle (2005a), only the main conclusions are reported in the following.

4.1 SOM surface of the hypoplastic model for clays Solution of Eqs. (4) and (5) for the hypoplastic model for clays is relatively straightforward, as the multiplier γ is independent of void ratio. This follows from the fact that for the HC model the Eq. (1) is for $\dot{f}_d = 0$ positively homogeneous of degree 1 with respect to **T** (the model assumes linear normal compression lines in the ln $p:\ln(1+e)$ space). Solution for the HC model reads

$$f_d = \|f_s \boldsymbol{\mathcal{A}}^{-1} : \mathbf{N}\|^{-1}$$
(7)

$$\vec{\mathbf{D}} = -\frac{\boldsymbol{\mathcal{A}}^{-1} : \mathbf{N}}{\|\boldsymbol{\mathcal{A}}^{-1} : \mathbf{N}\|}$$
(8)

with

$$\mathcal{A} = f_s \mathcal{L} + \frac{1}{\lambda^*} \mathbf{T} \otimes \mathbf{1}$$
(9)

The HC model assumes the following expression for the *pyknotropy* factor f_d :

$$f_d = \left(\frac{2p}{p_e^*}\right)^{\alpha} \tag{10}$$

where α is a scalar factor calculated from model parameters and p_e^* is Hvorslev's equivalent pressure at the isotropic normal compression line. (2). Therefore it is possible to calculate the value of p_e^* for given **T** (from (10) and (7))

$$p_e^* = -\frac{2}{3} \operatorname{tr} \mathbf{T} \| f_s \mathcal{A}^{-1} : \mathbf{N} \|^{1/\alpha}$$
(11)

The SOM surface of the HC model may be conveniently plotted in the normalised space \mathbf{T}/p_e^* .

The shape of the SOM surface of the HC model for four different sets of material parameters in the normalised space $p/p_e^*:q/p_e^*$ is plotted in Fig. 4, corresponding parameters are in Tab. 1 (London clay – Mašín 2005a; Beaucaire marl – Mašín et al. 2005; Bothkennar and Pisa clay – Mašín 2005b).



Figure 4. SOM surface of the hypoplastic model for clays for four different sets of material parameters.

 Table 1. Parameters of the hypoplastic model for clays

		71	1		5
soil	$\varphi_c [^\circ]$	λ^*	κ^*	N	r
Lond. c.	22.6	0.11	0.014	1.375	0.4
Beau. m.	33	0.057	0.007	0.85	0.4
Pisa c.	21.9	0.14	0.005	1.56	0.2
Both. c.	35	0.119	0.002	1.344	0.05

Table 2. Parameters of the hypoplastic model for granular materials for Zbraslav sand (Herle and Gudehus 1999)

$\varphi_c [^\circ]$	h_s [MPa]	n	e_{d0}
31	5700	0.25	0.52
e_{c0}	e_{i0}	α	eta

4.2 SOM surface of the hypoplastic model for granular materials

Explicit solution of Eqs. (4) and (5) is not available for the hypoplastic model for granular materials. In this case, the *pyknotropy* factor at SOM conditions reads

$$f_d = \left[\|\mathbf{B}\|^2 + \left(\frac{\|\mathbf{C}\| \left(\frac{1+e}{e}\right) \operatorname{tr} \mathbf{B}}{G - \left(\frac{1+e}{e}\right) \operatorname{tr} \mathbf{C}} \right)^2 + \frac{2(\mathbf{B} : \mathbf{C}) \operatorname{tr} \mathbf{B} \left(\frac{1+e}{e}\right)}{G - \left(\frac{1+e}{e}\right) \operatorname{tr} \mathbf{C}} \right]^{-\frac{1}{2}}$$
(12)

with

$$\mathbf{B} = \mathcal{L}^{-1} : \mathbf{N} \tag{13}$$

$$\mathbf{C} = \frac{\mathcal{L}^{-1} : \vec{\mathbf{T}}}{f_s} \tag{14}$$

$$G = \frac{n}{h_s} \operatorname{tr} \vec{\mathbf{T}} \left(\frac{3p}{h_s} \right)^{(n-1)}$$
(15)

Eq. (12) is an implicit equation for f_d , as the hypoplastic model for granular materials assumes

$$f_d = \left(\frac{e - e_d}{e_c - e_d}\right)^{\alpha} \tag{16}$$

and therefore

$$e = f_d^{(1/\alpha)}(e_c - e_d) + e_d$$
(17)

In addition, unlike in the case of the HC model, also factor f_s of the VW model is dependent on e. Eq. (12) may be, however, solved numerically.

Due to the particular formulation of factor f_d (16), different constant-volume cross-sections through the SOM surface have different shape (normalisation with respect to p_e^* is not applicable for the VW model). For Zbraslav sand parameters (Tab. 2) these cross-sections are shown in Fig. 5.



Figure 5. Constant-volume cross-sections through the SOM surface of the VW model for Zbraslav sand parameters.

5 META-STABLE STRUCTURE IN HYPOPLAS-TICITY

As mentioned in Sec. 4, Mašín and Herle (2005b) demonstrated that the swept-out-memory surface is a close approximation of the state boundary surface. Therefore, hypoplastic models should, in principle, allow us to modify the size of the state boundary surface in such a way that the models predict different stress-dilatancy behaviour of structured and reference materials, using the approach commonly applied in elasto-plastic models for structure soils (Sec. 3). The incorporation of meta-stable structure into two reference hypoplastic models will be discussed in this section.

5.1 Hypoplastic model for clays

Modification of the reference HC model for predictions of structured soils has been proposed by Mašín (2005b).

First, the reference model will be enhanced for predictions of the behaviour of soil with "stable" structure ($\dot{s} = 0$, $\dot{p}_t = 0$). A study of the expression for the SOM surface of the HC model (Sec. 4.1) reveals that the size of the SOM surface would be increased by the factor s by a simple replacement of p_e^* in the expression for f_d by sp_e^* :

$$f_d = \left(\frac{2p}{sp_e^*}\right)^{\alpha} \tag{18}$$

Effect of this modification on the SBS in the stressvoid ratio space is shown in Fig. 6.



Figure 6. SBS in the $\ln p : \ln(1+e)$ space for s > 1.

Further, the origin on the SOM surface would be shifted towards negative stresses along the isotropic axis if the Cauchy stress tensor **T** was in the constitutive model replaced by the "transformed stress tensor" \mathbf{T}^t (introduced already by Bauer and Wu (1994) who modified the early hypoplastic model considering **T** the only state variable)

$$\mathbf{T}^t = \mathbf{T} - p_t \mathbf{1} \tag{19}$$

Note that this is the most simple modification (constant p_t , $p_t < 0$), which implies that the SOM surface than does not have a unique image in the normalised space \mathbf{T}/p_e^* . Effect of the two modifications on the constant volume cross-section through the SOM surface of the HC model is demonstrated in Fig. 7.



Figure 7. Constant-volume cross-sections through the SOM surface of the HC model with stable structure (reference parameters for London clay).

Second, the meta-stable structure will be incorporated into the constitutive model by defining the evolution equations for additional state variables and by modifying the *barotropy* factor f_s . For simplicity only one additional state variable s will be considered ($p_t = 0$). Evolution equation for s may read (after Baudet and Stallebrass 2004)

$$\dot{s} = -\frac{k}{\lambda^*}(s - s_f)\dot{\epsilon}^d \tag{20}$$

where s_f is the ultimate value of s (typically for bonded materials $s_f = 1$), k is a parameter controlling the rate of structure degradation and $\dot{\epsilon}^d$ is a "damage" strain rate with formulation (Mašín 2005b)

$$\dot{\epsilon}^d = \sqrt{\left(\dot{\epsilon}_v\right)^2 + \frac{A}{1-A} \left(\dot{\epsilon}_s\right)^2} \tag{21}$$

 $\dot{\epsilon}_v$ and $\dot{\epsilon}_s$ are rates of the volumetric and shear strains respectively³ and A is a non-dimensional scaling parameter that controls their relative contribution to the structure degradation.

The *barotropy* factor f_s needs to be modified in order to ensure consistency in model predictions and the pre-defined structure degradation law (20). Formulation of the HC model for the isotropic compression from the isotropic normally compressed state reads

$$\dot{p} = -\left[\frac{1}{3(1+e)}f_s\left(3+a^2-2^{\alpha}a\sqrt{3}\right)\right]\dot{e} \quad (22)$$

The isotropic normal compression line of the model incorporating structure is given by (see Fig. 6)

$$\ln(1+e) = N + \lambda^* \ln s - \lambda^* \ln\left(\frac{p}{p_r}\right)$$
(23)

Time differentiation of (23) results in

$$\frac{\dot{e}}{1+e} = \lambda^* \left(\frac{\dot{s}}{s} - \frac{\dot{p}}{p}\right) \tag{24}$$

The isotropic formulation of the structure degradation law (20-21) is

$$\dot{s} = \frac{k}{\lambda^*} (s - s_f) \frac{\dot{e}}{1 + e} \tag{25}$$

Combination of (24) and (25) yields

$$\frac{\dot{p}}{p} = -\left[\frac{s-k(s-s_f)}{\lambda^* s}\right]\frac{\dot{e}}{1+e}$$
(26)

³In hypoplasticity, Eq. (21) is defined in terms of total, instead of plastic strain rates. The difference between the two definitions is in the large-strain range insignificant, as high "truly" elastic stiffness causes the elastic part of the total strain increment to be negligible with respect to the plastic part. which may be compared with (22) to find an expression for the *barotropy* factor f_s of the new hypoplastic model:

$$f_s = \frac{3p}{\lambda^* s} \left[s - k(s - s_f) \right] \left(3 + a^2 - 2^\alpha a \sqrt{3} \right)^{-1}$$
(27)

The influence of parameter k on predictions of an isotropic compression test is shown in Fig. 8, evaluation of model predictions in Sec. 6.



Figure 8. Prediction of an isotropic compression test by the structured HC model, influence of parameter k (parameters for Pisa clay).

5.2 Hypoplastic model for granular materials

Hypoplastic model for granular materials may be modified to incorporate the effects of inter-particle bonding in a similar way as the HC model. It is, however, worth noting that bonding in granular materials influences the small-strain behaviour more significantly than structure effects in clays (by significant enlargement of the elastic range – see, e.g., Coop and Atkinson 1993). Therefore, the VW model for structured soils will in its basic version necessarily performs poorly in the small- to medium-strain range and additional measures (e.g. on the basis of the intergranular strain concept) are needed to ensure its practical applicability.

It follows from Sec. 4.2 that the size of the SBS of the VW model may be increased by modifying the characteristic void ratios e_{i0} , e_{c0} and e_{d0} . Their increase by a factor s^e in the form

$$e_{x0s} = e_{x0} + s^e (28)$$

where x stands for i, c, d and e_{x0s} are characteristic void ratios of a structured soil at p = 0 kPa, would have the influence on the size of the SBS as demonstrated in Fig. 9.

Characteristic void ratios e_{is} , e_{ds} and e_{cs} are now calculated by

$$e_{xs} = (e_{x0} + s^e) \exp\left[-\left(\frac{-\text{tr}\mathbf{T}}{h_s}\right)^n\right] \qquad (29)$$



Figure 9. Constant-volume cross-sections through the SOM surface of the VW model for reference $(s^e = 0)$ and structured $(s^e = 0.1)$ soil (reference parameters for Zbraslav sand).

where e_{xs} comes in place of e_x in the expression of factors f_d and f_s . Note that e_{xs} are *not* the state limits of the structured soil, as the evolution equation for s^e is a natural component of the modified constitutive equation. Similarly to the HC model, the SBS may be further shifted by introducing the transformed stress tensor \mathbf{T}^t .

The evolution equation for s^e may read (similarly to the structured HC model)

$$\dot{s}^e = -k^e (s^e - s^e_f) \dot{\epsilon}^d \tag{30}$$

where typically $s_f^e = 0$ for bonded material, $\dot{\epsilon}^d$ may be calculated according to (21), k^e controls the rate of the structure degradation.

Finally, the factor f_s must be modified to ensure that the consistency condition at the isotropic normally compressed state is not violated. Formulation of the VW model for the isotropic compression from the isotropic normally compressed state is given by

$$\dot{p} = \frac{-f_s}{3(1+e)} \left[3 + a^2 - a\sqrt{3} \left(\frac{e_{i0} - e_{d0}}{e_{c0} - e_{d0}} \right)^{\alpha} \right] \dot{e} \quad (31)$$

The isotropic normal compression line of the model

incorporating structure reads

$$e = (e_{i0} + s^e) \exp\left[-\left(\frac{-\text{tr}\mathbf{T}}{h_s}\right)^n\right]$$
(32)

Time differentiation of (32) results in

$$\frac{\dot{e}}{e} = \frac{\dot{s}^e}{e_{i0} + s^e} - n\frac{3\dot{p}}{h_s} \left(\frac{3p}{h_s}\right)^{n-1}$$
(33)

The isotropic formulation of the structure degradation law (30) and (21) is

$$\dot{s}^e = k^e (s^e - s^e_f) \frac{\dot{e}}{1+e}$$
 (34)

Combination of (33) and (34) yields

$$n\frac{3\dot{p}}{h_s}\left(\frac{3p}{h_s}\right)^{n-1} = \left[\frac{k^e(s^e - s_f^e)}{(1+e)\left(e_{i0} + s^e\right)} - \frac{1}{e}\right]\dot{e} \quad (35)$$

The factor f_s of the structured VW hypoplastic model follows from the comparison of (31) and (35) and unlike the factor f_s of the HC model, it contains also the *pyknotropy* component $(e_{is}/e)^{\beta}$ (for details see Gudehus 1996).

$$f_s = \left(\frac{e_{is}}{e}\right)^{\beta} \left[\frac{1+e_{is}}{e_{is}} - \frac{k^e \left(s^e - s_f^e\right)}{e_{i0} + s^e}\right]$$
$$\frac{h_s}{n} \left(\frac{3p}{h_s}\right)^{1-n} \left[3+a^2 - a\sqrt{3} \left(\frac{e_{i0} - e_{d0}}{e_{c0} - e_{d0}}\right)^{\alpha}\right]^{-1} \tag{36}$$

6 EVALUATION

Thorough evaluation of the structured HC model is given in Mašín (2005b). In Figs. 10, 11 and 12 are shown predictions of the experiments on natural Pisa clay reported by Callisto and Calabresi (1998). All the parameters of the structured HC model except the parameters that quantify the effects of structure (k, A)and s_f) were calibrated solely on the basis of experiments on reconstituted clay. Only two experiments (A0 and A90, for normalised stress path see Fig. 10) were used for calibration of parameters k, A and s_f . Although the model was calibrated using a simple procedure with a minimal number of laboratory experiments required, its predictions are satisfactory in the entire range of stress paths directions. Parameters of the structured HC model for natural Pisa clay are in Tab. 3.

7 CONCLUSIONS

The paper presented a conceptual framework for incorporation of meta-stable structure into hypoplastic constitutive models. The approach is based on the

Table 3. Parameters of the structured HC model for natural Pisa clay

$$\begin{array}{|c|c|c|c|c|c|c|c|} \varphi_c & \lambda^* & \kappa^* & N & r & k & A & s_f \\ \hline 21.9^\circ & 0.14 & 0.005 & 1.56 & 0.2 & 0.4 & 0.1 & 1 \\ \end{array}$$



Figure 10. Normalised stress paths of the natural and reconstituted Pisa clay (data from Callisto and Calabresi 1998) and predictions by the structured HC model (from Mašín 2005b).

modification of the state boundary surface predicted by the models. By increasing the size of the state boundary surface it is possible to take into account different stress-dilatancy behaviour of structured and reference soils.

The approach is equivalently applicable to the two reference hypoplastic models. Predictive capabilities of the structured hypoplastic model for clays were demonstrated, structured hypoplastic model for granular materials, however, requires further modification in order to take into account the large quasi-elastic range caused by inter-particle bonding.

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Figure 11. Experiments on natural Pisa clay plotted in the $\ln(p/p_r) : \ln(1+e)$ space (data from Callisto and Calabresi 1998) and predictions by the structured HC model (from Mašín 2005b).

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Figure 12. (a) $\epsilon_s : q$ curves from experiments on natural Pisa clay (data from Callisto and Calabresi 1998) and predictions by the structured HC model (from Mašín 2005b).

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