CALIBRATION OF AN ADVANCED SOIL CONSTITUTIVE MODEL FOR USE IN PROBABILISTIC NUMERICAL ANALYSIS

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ABSTRACT: *In this paper, a horizontally stratified deposit of a sandy soil is analysed using a probabilistic approach. The mechanical behaviour of the soil is described by an advanced hypoplastic model for sand and by the basic Mohr-Coulomb model. The models are calibrated using an automated procedure based on a set of experimental data on forty specimens recovered from a quarry wall in a regular grid. It is shown that in order to describe the soil deposit properly using probabilistic approach, spatial variability of soil parameters (not state variables) needs to be considered. Experimental data also allowed us to quantify the spatial correlation structure (measured by the correlation length* θ*), which is significantly higher in horizontal direction (*$\theta_h = 242$ *m) than in vertical direction (*$\theta_v = 5.1$ *m). The results may be used as an input into numerical methods based on random field theories.*

1 INTRODUCTION

Geomaterials feature high spatial variability, caused by the processes by which they were formed and which they underwent in the past. In addition to inherent spatial variability of soil properties, the geotechnician must cope with insufficient amount of experimental data and with inaccuracies of measurements. For these reasons, probabilistic approach is well suited for use in geotechnical engineering applications.

A rational means of quantification of soil spatial variability is to model the distribution of soil mechanical properties as homogeneous random fields, in which deviation of a given property from the trend value is characterised using some suitable *statistical distribution*. Spatial variability is measured by means of the *correlation length* θ, which describes the distance over which the spatially random values tend to be significantly correlated (Vanmarcke 1983).

Many examples of the use of probabilistic methods in geotechnical engineering are now available throughout literature, but the researchers often use only simple constitutive models (typically, Mohr-Coulomb model). Applications of probabilistic methods with advanced constitutive models are limitted to considering spatial variability of *state variables*, with model *parameters* often treated as deterministic. For example, Hicks & Onisiphorou (2005) studied stability of underwater sandfill berms. They used a double-hardening constitutive model Monot by Molenkamp (1981) with probabilistic distribution of state parameter ψ. As their aim was to study whether presence of 'pockets' of liquifiable material may be enough to cause instability in a predominantly dilative fill, using probabilistic distribution of state parameter only is in this case fully justified. In other applications, Tejchman (2006) studied the influence of the fluctuation of void ratio on formation of the shear zone in the biaxial specimen using the hypoplastic model by von Wolffersdorff (1996). The same constitutive model with spatially variable void ratio used Niemunis et al. (2005) and Nübel & Karcher (1998) in FE simulations of different geotechnical problems. Finally, Andrade et al. (2008) considered random porosity fields in combination with a constitutive model by Borja & Andrade (2006) and studied their influence on strength and shear band formation in a biaxial specimen .

Though considering spatial variability of state variables only is often justifiable approach, it does not provide a complete description of soil variability. Many soil deposits are horizontally stratified with variable granulometry and grain mineralogy in different layers. Advanced constitutive models are constructed in such a way that the soil parameters are independent of soil state (quantified by stress tensor and void ratio), they depend on grain size, shape and mineralogy only. Therefore, spatial variability of parameters should be considered in these cases. Detailed quantification of the spatial variability in horizontally stratified deposits is the purpose of this paper.

2 TEST MATERIAL

The material for investigation comes from the south part of upper Cretaceous Třeboň basin in the South Bohemia from the sand pit ”Kolný”. The pit is located in the upper part of the so-called Klikovské layers, youngest (senon) strata of south Bohemian basins. These fluvial layers are characterised by a rhythmical variation of gravely sands, sands and sands with dark grey clay inclusions.

For the purpose of this work, altogether forty samples were taken from a ten meters high pit wall (Fig. 1). The samples were taken in four horizontal levels. The vertical distance between sampling levels was 3 meters, horizontal distance between sampling points was 4 meters. Depth of the upper sampling level was 1 m below the ground surface; the bottom level was therefore in the depth of 10 m.

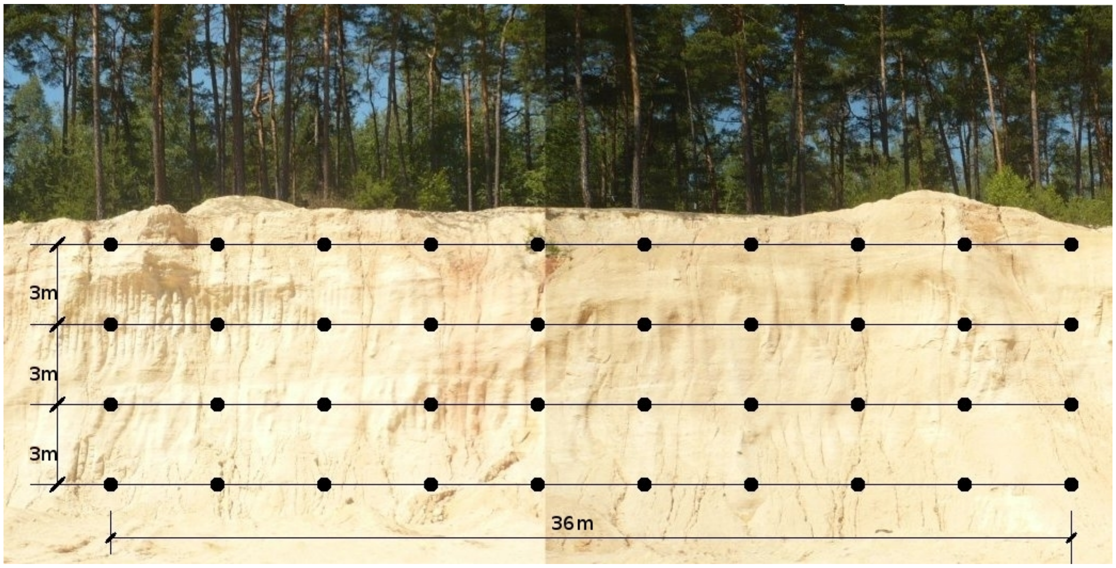


Fig. 1. The wall of the sand pit in south part of the Třeboň basin. Black dots represent positions of specimens taken for the laboratory investigation.

Five *in-situ* porosity tests with membrane porosimeter were performed at different locations within the area from which the samples were obtained. Natural void ratios were 0.267, 0.289, 0.308, 0.298 and 0.302. The porosity is therefore fairly uniform and the sand is in very dense conditions. These low porosity values are caused by a few factors. First, the soil is well graded with a wide granulometry curve. The second factor is the presence fines with grain size below 0.063 mm. The amount of fines is different in different layers with the maximum percentage of about 7%.

3 LABORATORY PROGRAM AND CALIBRATION OF CONSTITUTIVE MODELS

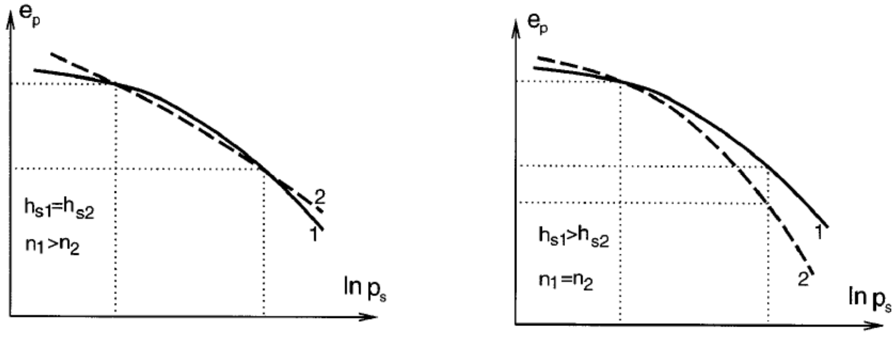
The laboratory program was selected to provide for each of the samples enough information to calibrate a hypoplastic model for granular materials by von Wolffersdorff (1996) and Mohr-Coulomb constitutive model. The following tests were performed on each of the 40 samples:

- Oedometric compression test on initially very loose specimens with loading steps 100, 200, 400, 800, 1600, 3200 and 6400 kPa.
- Drained triaxial compression test on specimen dynamically compacted to void ratio corresponding to the dense *in-situ* conditions. One test per specimen at the cell pressure of 200 kPa.
- Measurement of the angle of repose.

The whole process of calibration was automated using Linux scripting language Bash in combination with text manipulation language AWK.

3.1 Calibration of the hypoplastic constitutive model for granular materials

The hypoplastic model by von Wolffersdorff (1996) has eight material parameters. The model was calibrated using procedures outlined by Herle & Gudehus (1999). The critical state friction angle ϕ_c was obtained directly by the measurement of the angle of repose. The next two parameters h_s and n were directly computed from oedometric loading curves in the interval of $\sigma_a \in \langle 100, 1000\rangle$ kPa. The parameter n controls the curvature of oedometric curve and h_s controls the overall slope of oedometric curve as is shown in Figs 2 (a) and (b).



(a) (b)

Fig. 2. Influence of n (a) and h_s (b) on calculated oedometric curves (Herle & Gudehus 1999).

The parameter n was computed from

$$n = \frac{\ln(e_{p1}C_{c2}/e_{p2}C_{c1})}{\ln(p_{s2}/p_{s1})} \tag{1}$$

where mean stresses p_{s1} and p_{s2} were calculated from axial stresses using the Jáky formula $K_0 = 1 - \sin\phi_c$, and e_{p1} and e_{p2} are the void ratios corresponding to stresses p_{s1} and p_{s2}. Tangent compression indices corresponding to the limit values of the interval p_{s1} and p_{s2} (C_{c1} and C_{c2}) were approximated by secant moduli between loading steps preceding and following the steps p_{s1} and p_{s2}. The parameter h_s was obtained from

$$h_s = 3p_s \left(\frac{ne_p}{C_c}\right)^{1/n} \tag{2}$$

where C_c is a secant compression index calculated from limit values of the calibration interval p_{s1} and p_{s2}; p_s and e_p are averages of the limit values of p and e for this interval.

Following Herle & Gudehus (1999), initial void ratio e_{max} of a loose oedometric specimen was considered equal to the critical state void ratio at zero pressure e_{c0}. Figure 3 shows comparison of compression curves calculated using formula

$$e = e_{c0}\ \exp\left[-\left(\frac{3p}{h_s}\right)^n\right] \tag{3}$$

with compression curves obtained from the oedometric test (for illustration purposes one experiment from each sampling level only), demonstrating the the values of h_s, n and e_{c0} obtained using the outlined procedures represent well the oedometric compression curves.

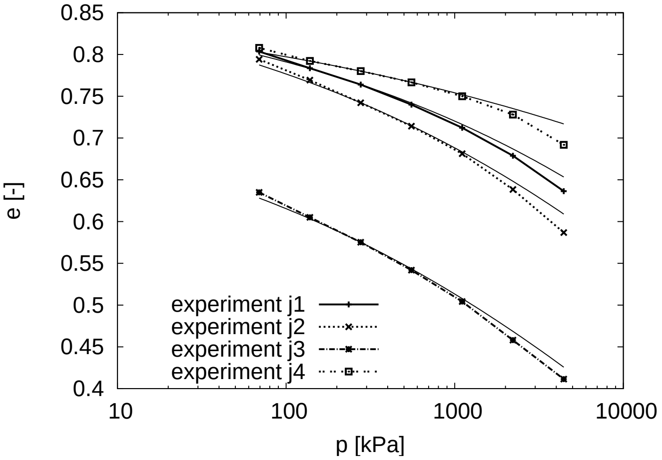


Fig. 3. Computed curves using h_s, n, e_{c0} parameters for one column of specimens.

Void ratios e_{d0} and e_{i0}, which are the next two parameters, were obtained from empirical relations. The physical meaning of e_{d0} is void ratio at maximum density, void ratio e_{i0} represents intercept of the isotropic normal compression line with $p = 0$ axis. Void ratio e_{i0} was obtained by multiplication e_{c0} by a factor 1.2. The ratio $e_{i0}/e_{c0} \approx 1.2$ was derived by Herle & Gudehus (1999) considering skeleton consisting of ideal spherical particles.

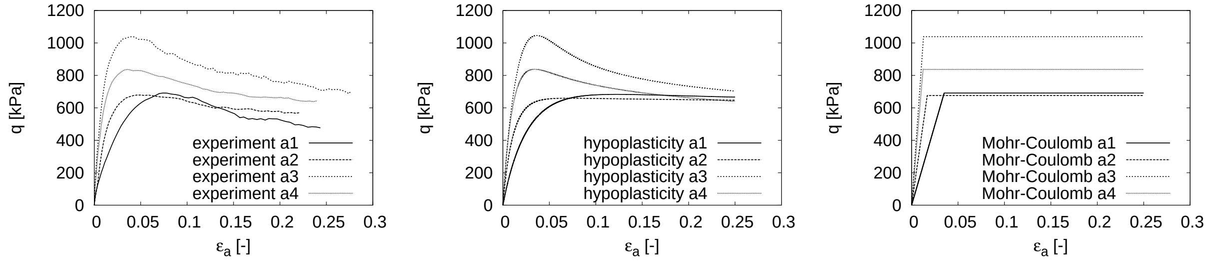
The minimum void ratio e_{d0} was also calculated from e_{c0} by its multiplication by a factor f_{ed}. In estimating f_{ed} we recalled that the *in-situ* state and initial state of triaxial specimens were very dense. The ratio e_0/e_c was calculated for each of the 40 triaxial specimens (where e_0 is the initial void ratio of the triaxial tests, e_c is the value of e calculated using Eq. (3)). The factor f_{ed} was than considered equal to the minimum value of e_0/e_c from all 40 specimens (namely,

$f_{ed} = 0.379$). In this way it was ensured that the initial void ratio for triaxial specimens is always higher than e_d and the initial state is close to the state of maximum density, which corresponds to the *in-situ* conditions.

The last two parameters α and β were calibrated by single-element simulations of the drained triaxial tests. The two parameters control independently different aspects of soil behaviour, namely the parameter β controls the shear stiffness and α controls peak friction angle. For calibration of β, its value was varied in the interval $\beta \in (0.1, 3)$ with increment $\varDelta\beta = 0.1$. The triaxial test was simulated for each value of β and experimental and simulated axial strains (ϵ_a^e and ϵ_a^s) corresponding to 70 % of maximum experimental stress deviator q were compared. β corresponding to the minimum value of $|\epsilon_a^e - \epsilon_a^s|$ was considered as the most satisfactory.

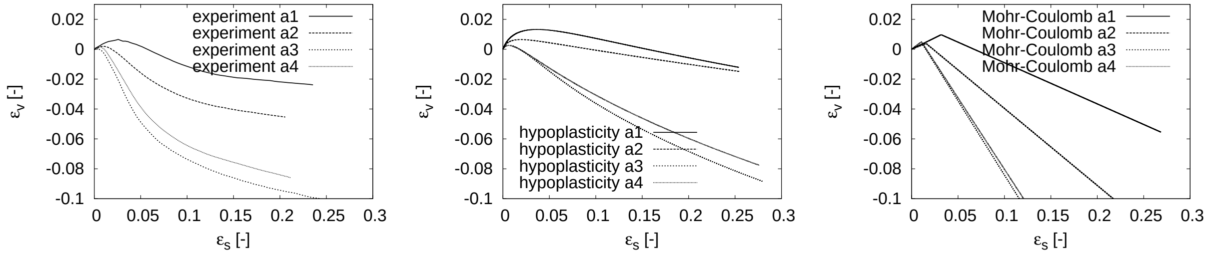
The parameter α controls peak friction angle. For calibration of α similar method like for the parameter β was used. Parameter α was varied in the interval $\alpha \in (0.01, 0.3)$ with increment of $\varDelta\alpha = 0.01$, maximum experimental and simulated deviator stresses (q_{max}^e and q_{max}^s) were compared and α corresponding to the minimum $|q_{max}^e - q_{max}^s|$ was considered as the most appropriate.

Figs. 4 and 5 show comparison of typical experimental and simulated q vs. ϵ_a and ϵ_v vs. ϵ_s curves (one specimen for each sampling level). The hypoplastic model calibrated using the outlined procedures reproduces closely the q vs. ϵ_a curves and it underestimates the rate of dilatancy.



(a) (b) (c)

Fig. 4. Typical experimental q vs. ϵ_a curves (a) compared with predictions by the hypoplastic (b) and Mohr-Coulomb (c) constitutive models, one specimen for each sampling level.



(a) (b) (c)

Fig. 5. Typical experimental ϵ_v vs. ϵ_s curves (a) compared with predictions by the hypoplastic (b) and Mohr-Coulomb (c) constitutive models, one specimen for each sampling level.

3.2 Calibration of the Mohr-Coulomb constitutive model

The Mohr-Coulomb model has five parameters ϕ, E, ν, ψ and c. The Young modulus E was calculated as secant modulus for 50 % peak deviator stress q. The friction angle ϕ was calibrated to represent the peak state. In the calibration, zero cohesion c was considered. It could not be evaluated as only one triaxial test was performed for each specimen. The q vs. ϵ_a curves simulated with parameters E and ϕ obtained using the described procedure are shown in Fig. 4 (c).

Poisson ratio ν was counted from the initial (contractive) portion of the ϵ_a vs. ϵ_v curve using equation (4), where $\mathit{\Delta}\epsilon_a$ and $\mathit{\Delta}\epsilon_v$ are the axial and volumetric strains at maximum contraction:

$$\nu = \frac{\mathit{\Delta}\epsilon_a - \mathit{\Delta}\epsilon_v}{2\mathit{\Delta}\epsilon_a} \tag{4}$$

The dilatancy angle ψ was calculated from

$$\frac{\mathit{\Delta}\epsilon_v}{\mathit{\Delta}\epsilon_s} = \frac{6\mathrm{sin}\psi}{3\mathrm{sin}\psi - 1} \tag{5}$$

to represent as closely as possible the initial more-or-less linear dilatant portion of the ϵ_s vs. ϵ_v curve (see Fig. 5 (c)).

4 PROBABILISTIC DISTRIBUTION OF MODEL PARAMETERS

Statistical distribution of parameters for all 40 specimens is shown in Fig. 6 for the hypoplastic model and in Fig. 7 for the Mohr-Coulomb model. Parameters e_{i0} and e_{d0} of the hypoplastic model are not presented in Fig. 6 as they are multiples of the value of e_{c0}.

Suitablity of normal and log-normal distributions to represent the experimental data was studied using Kolmogorov-Smirnov and χ^2 tests. All the parameters except the parameter h_s and α of the hypoplastic model are best represented by the Gaussian distribution, parameters h_s and α follow log-normal distribution. The Gaussian or log-normal fits may be considered as reasonable, but not prefect, partly due to the inherent properties of the models and partly due to the lack of experimental data. Means and standard deviations for the normally distributed parameters and medians and modes for the log-normally distributed parameters are summarised in Tab. 1 for the hypoplastic model and Tab. 2 for the Mohr-Coulomb model. Notice that the deviation of the parameter values from the mean is relatively high. This effect could be reproduced by considering fluctuation of void ratio only.

5 EVALUATION OF CORRELATION LENGTH

Estimation of the correlation length θ based on experiments presented in this paper is described in this section. As the sediments of the Cretaceous basin tested in this work are fluvial, they tend to be horizontally stratified. Therefore, it may be expected that the correlation length will be higher in horizontal direction (θ_h) than in vertical direction (θ_v).

The correlation coefficient $\rho_{X,Y}$ between two variables X and Y is defined as

$$\rho_{X,Y} = \frac{E\left[(X-\mu_X)(Y-\mu_Y)\right]}{\sigma_X \sigma_Y} \tag{6}$$

where E is the expected value operator (mean), μ and σ represent mean and standard deviation respectively. The values of ρ for the critical state friction angle ϕ_c calculated for specimens

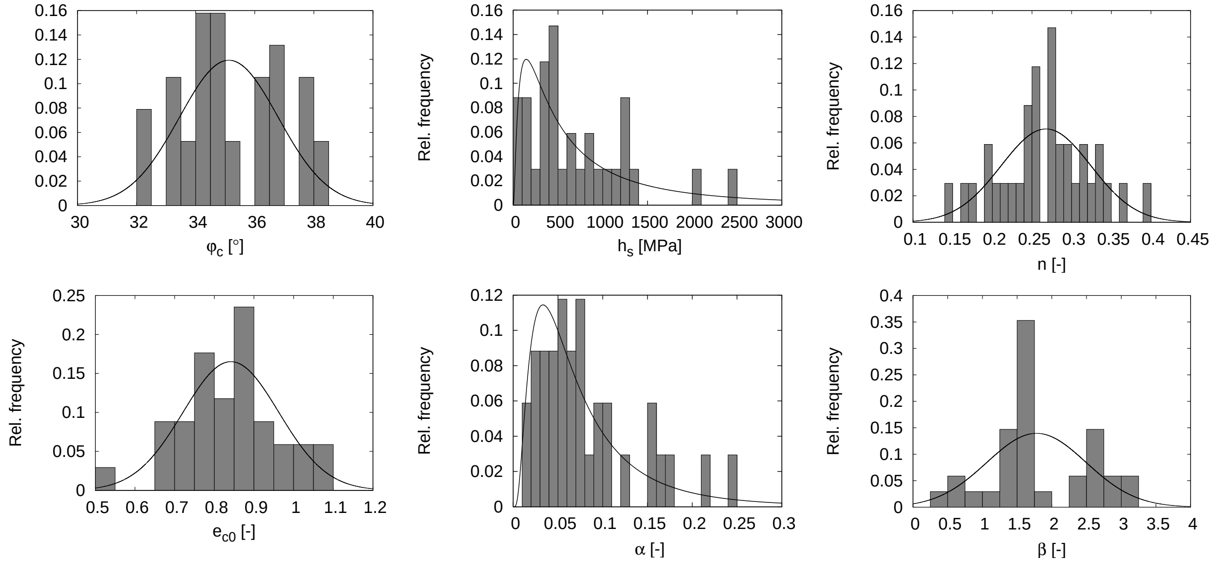


Fig. 6. Statistical distribution of parameters of the hypoplastic model for all 40 specimens.

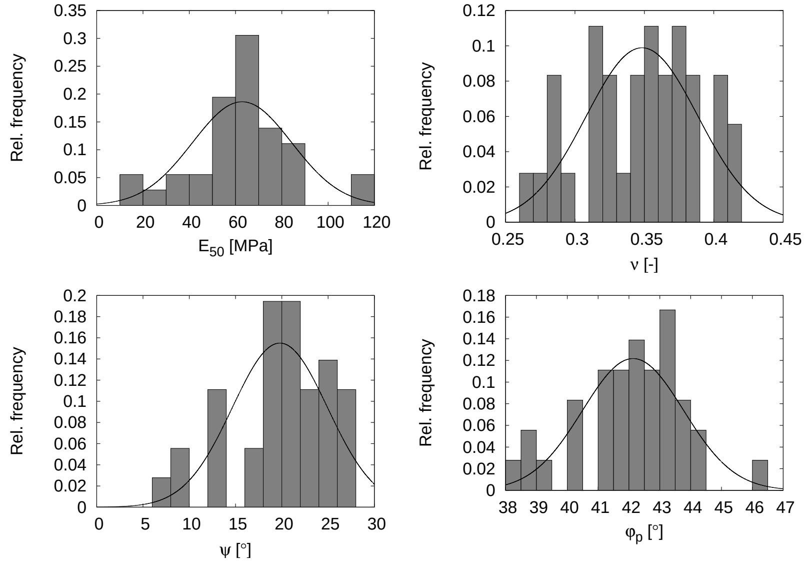


Fig. 7. Statistical distribution of parameters of the Mohr-Coulomb model for all 40 specimens.

in different columns of the sampling grid are shown in Fig. 8a with respect to the horizontal distance between the columns. Fig. 8b shows correlation coefficient between different sampling levels as a function of the sampling level distance. Clearly, correlation in the vertical direction is poor, whereas correlation in the horizontal direction is very good. This agrees with the horizontal stratification of the deposit. As ϕ_c measured as an angle of repose is independent of soil state, it also demonstrates a need for considering spatial variability of model parameters (not only state variables) in probabilistic simulations.

Table 1. Characteristic values of statistical distributions of parameters of the hypoplastic model.

normal distribution	mean	standard deviation
ϕ_c	35.1°	1.69°
n	0.268	0.057
e_{c0}	0.842	0.121
e_{i0}	1.010	0.145
e_{d0}	0.319	0.046
β	1.779	0.715
lognormal distribution	**median**	**mode**
α	0.063	0.035
h_s	564 MPa	145 MPa

Table 2. Characteristic values of statistical distributions of parameters of the Mohr-Coulomb model.

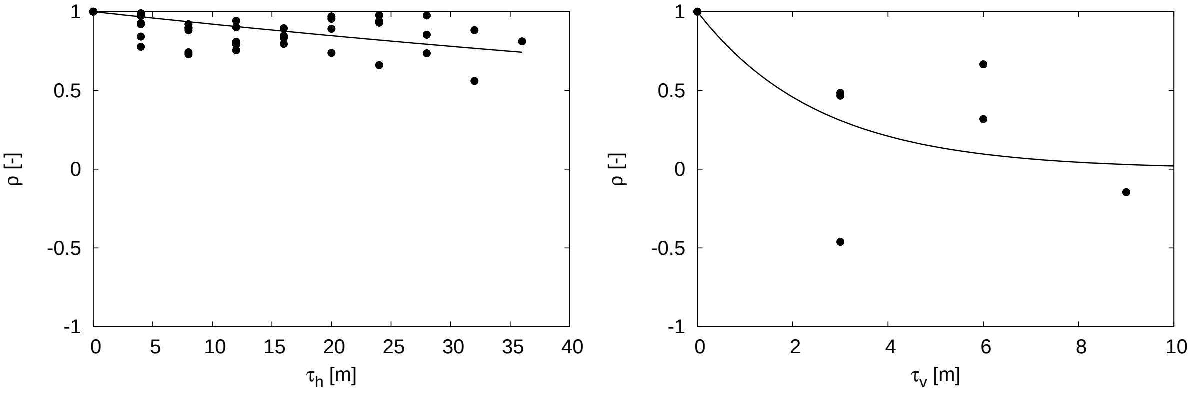
normal distribution	mean	standard deviation
ϕ	42.1°	1.64°
E	62.8 MPa	21.4 MPa
ν	0.348	0.040
ψ	19.8°	5.15°

To evaluate the correlation lengths θ_h and θ_v, the dependency of the correlation coefficient ρ on distance was approximated using exponential equation due to Markov

$$\rho = \exp\left[-2\sqrt{\left(\frac{\tau_h}{\theta_h}\right)^2 + \left(\frac{\tau_v}{\theta_v}\right)^2}\right] \tag{7}$$

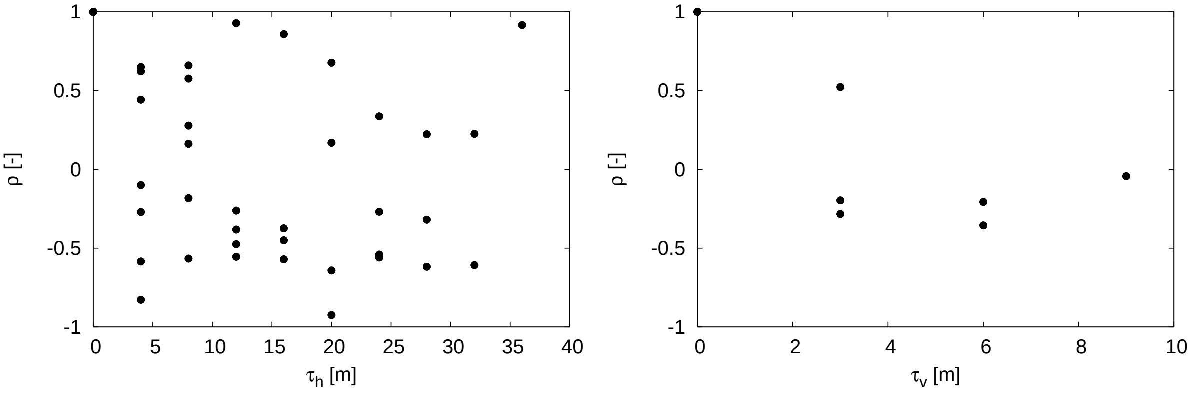
where τ_h is the horizontal distance between two specimens and τ_v is the vertical distance. The least square fit of Eq. (7) through the experimental data is also shown in Fig. 8, leading to $\theta_h = 242$ m and $\theta_v = 5.1$ m. Note that practically no correlation is observed in the vertical direction, therefore the obtained value $\theta_v = 5.1$ m is implied by the adopted vertical sampling distance, rather than by the actual correlation properies.

Critical state friction angle ϕ_c was measured as an angle of repose, it thus represents a simple means of quantification of inherent soil property independent of soil state. In principle, all parameters of the hypoplastic model should have the same character – they should be independent of soil state, which is quantified by the stress tensor and void ratio. For this reason, spatial correlation length should be quantifiable using any from the hypoplastic parameters. In the present study, however, this approach could not be used successfully. Evaluation of the dependency of the correlation length on sampling distance using other parameters than ϕ_c leads to highly scattered data without clear trend. As an example, see Fig. 9 for the parameter h_s. The same applies to the parameters of the Mohr-Coulomb model. Regardless this fact, however, we presume that the spatial correlation evaluated using the directly-measured value of ϕ_c provides a reasonable approximation of the correlation length for other soil parameters.



(a) (b)

Fig. 8. Evaluation of the correlation coefficient ρ in horizontal (a) and vertical (b) directions for parameter ϕ_c, together with least square fit of Eq. (7).



(a) (b)

Fig. 9. Evaluation of the correlation coefficient ρ in horizontal (a) and vertical (b) directions for parameter h_s, showing no clear trend with distance.

6 CONCLUSIONS AND OUTLOOK

The research project, whose part was presented in this paper, aims at quantification of spatial variability in horizontally stratified deposit of a sandy soil. The mechanical behaviour of sand was characterised using an advanced hypoplastic model and using a basic Mohr-Coulomb model for comparison. We demonstrated that in order to characterise the soil deposit using probabilistic approach properly, variability of soil parameters (not state variables) needs to be considered. Based on a comprehensive set of experimental data and automated calibration procedure, statistical distributions of parameters of the two constitutive models were evaluated. In addition, known original locations of the tested specimens allowed us to quantify the spatial correlation structure of soil parameters. Based on the value of the critical state friction angle ϕ_c, which is an inherent soil property, we have shown that the spatial correlation is high in horizontal direction ($\theta_h = 242$ m) and poor in vertical direction ($\theta_v = 5.1$ m).

The research will continue by evaluation of the influence of the spatial variability on results of simulations of typical geotechnical problems. Results obtained using random field finite element method will be compared with deterministic simulations based on average material properties.

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