Modelling of the Lodalen slide using probabilistic numerical methods

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ABSTRACT: A particular landslide in the fine-grained soil, Lodalen slide, has been simulated using finite element method combined with random field theory and Monte Carlo method. Parameters c and φ of the Mohr-Coulomb model have been considered as uncorrelated random variables. The calculated probability of failure is influenced by the correlation length θ, a parameter difficult to evaluate from geotechnical site investigation data. Not considering the spatial variability of soil properties would lead to unconservative design. In the presented case a simpler probabilistic method with infinite θ and variance reduced due to spatial averaging along the slip surface may be used succesfully, this result however does not have a general validity.

1 INTRODUCTION

The soil mechanical properties obtained from detailed geotechnical site investigations show a marked dispersion, coming from their inherent spatial variability (even in zones which are often regarded as "homogeneous" from the deterministic point of view) and measurement error. Additional uncertainty is introduced by the fact that only limitted number of measurements is often available, and from subjective calibration of simple constitutive models, which are often used in geotechnical analyses. These uncertainties are in geotechnical engineering commonly treated using deterministic concept of factors, which however discourage clearer understanding of the relative importance of the various factors involved (Singh and Chung 1991; Phoon and Kulhawy 1999). In this respect, probabilistic approaches are well suited to geotechnical engineering. Their rather limitted use in practical applications is caused by the lack of data needed for detailed statistical evaluation of mechanical properties.

Considering for the moment the inherent spatial variability only, it can be decomposed into smoothly varying trend function $t(z)$ and a fluctuating component $w(z)$ as follows:

$$\xi(z) = t(z) + w(z) \tag{1}$$

in which ξ is the in situ soil property and z is the depth (Phoon and Kulhawy 1999), see Fig. 1. A rational means of quantifying inherent variability is to model $w(z)$ as a homogeneous random field, in which deviation of $\xi(z)$ from the trend value is characterised

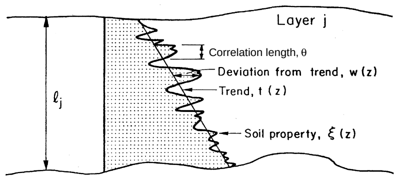

Figure 1. Characterisation of inherent soil variability (after Phoon and Kulhawy 1999, modified).

using some suitable statistical distribution, and spatial variability is measured by means of the correlation length θ, which describes the distance over which the spatially random values will tend to be significantly correlated (Vanmarcke 1983). Evaluation of the last quantity, θ, is in geotechnical engineering particularly difficult, as it requires vast amount of data, which are usually not available. Moreover, θ in horizontal direction (θ_h) is larger than the vertical (θ_v) in the case of horizontally stratified soil deposits. Detailed literature reviews on the values of correlation lengths present Phoon and Kulhawy (1999) and El-Ramly et al. (2003). It is observed that θ_h vary within the range 10-40 m, while θ_v ranges from 0.5 to 3 m.

The aim of this paper is to investigate implications of uncertainty in the correlation length values on the probability of failure of a slope computed by means of a deterministic numerical method combined with random field theory. A well-documented case history, namely slide in Lodalen, Norway (Sevaldson 1956), is used for the purpose of this evaluation.

2 PROBABILISTIC NUMERICAL METHODS

In probabilistic numerical analyses we usually need to evaluate statistical distribution of a performance function, based on known statistical characteristics of input variables. In the following, we will distinguish two classes of probabilistic methods. Methods of the first class ignore the random spatial structure of input variables, in other words they assume infinite correlation length θ. Methods of the second class are based on random field theories and consider spatial variability of input variables. Obviously, the first class models are special cases of the second class models for infinite θ.

A number of methods of the first class have been used in geotechnical applications. In this class, approximate analytical solutions, such as the First order second moment method as a special example of Taylor's series method (Duncan 2000) or point estimate methods (Christian and Baecher 1999), are available. These methods are popular as they require low number of computer runs. They, however, always consider a number of simplifying assumptions, so carefull analysis is needed to show wheather these assumptions are justifiable. An alternative to the analytical methods is the Monte Carlo simulation (or other advanced probability procedures, such as Latin Hypercube sampling), which are fully general, but depending on the problem solved they may require a significantly large number of realisations and consequently a considerable computational effort. Having the performance function $Y = g(X_1, X_2, ...X_n)$ of n independent random variables X_i, the difference ϵ of the true mean value of Y and mean value estimated using the Monte Carlo approach may be for normally distributed Y found from the Chebychev inequality:

$$P\left(\epsilon \leq \frac{\sigma[Y]}{\sqrt{m(1-\alpha)}}\right) \geq \alpha \tag{2}$$

where $\sigma[Y]$ is the standard deviation of the performance function, m is a number of realisations and α is a prescribed probability of accuracy of the estimate.

Disadvantage of the first class methods is that they ignore the spatial variability of soil properties. As demonstrated by many authors (Popescu et al. 1997, Haldar and Babu 2007, Hicks and Onisiphorou 2005, Griffiths and Fenton 2004), the spatial variability (and consequent concentration of less competent materials into distinct zones), may lead to a significant increase of the probability of unsatisfactory performance. This shortcoming may be overcome by the second class models. In their case the analytical evaluation is not possible, so the problem must be solved using iterative probabilistic methods, such as Monte Carlo, Latin Hypercube sampling, etc. The spatial autocorrelation of soil properties enters the calculation through the distance-dependent correlation coefficient ρ, described commonly by the exponential equation due to Markov:

$$\rho = \exp\left(\frac{-2\tau}{\theta}\right) \tag{3}$$

where τ is absolute distance between two points in a random field. Eq. 3 may be readily modified for $\theta_x \neq \theta_y$:

$$\rho = \exp\left(-2\sqrt{\left(\frac{\tau_x}{\theta_x}\right)^2 + \left(\frac{\tau_y}{\theta_y}\right)^2}\right) \tag{4}$$

where τ_x and τ_y are distances between two points in horizontal and vertical directions respectively. Random fields of input variables may be generated using one of a number of methods available (see Fenton (1994) for an overview). In the present contribution, a simple method based on Cholesky decomposition of the correlation matrix (e.g., Fenton (1997)) is used. This method is prone to numerical round-off errors when the number of points in the field becomes large, it is however sufficient for the present application.

When the random field models are used in continuum numerical methods with finite size of material domains, the point statistics of random input variables must be transformed through local spatial averaging about the domain size. In the case of normally distributed random variables, the mean remains unaffected, the standard deviation is reduced by:

$$\gamma = \left(\frac{\sigma[X_i]_A}{\sigma[X_i]}\right)^2 \tag{5}$$

where $\sigma[X_i]$ describes the point statistics of the variable X_i and $\sigma[X_i]_A$ is the standard deviation of the spatially averaged field. The variance reduction factor γ is calculated by integration of the Markov function (Eq. 4), see Vanmarcke (1983).

Some authors (Christian et al. 1994, Schweiger and Peschl 2005) advocate to use the idea of variance reduction due to spatial averaging also in combination with the first class models (models with homogeneous fields of random variables), in a way in which the standard deviation of the homogeneous field of random variables is reduced by Eq. (5) due to spatial averaging along the potential failure surface. This method will be in this paper denoted as extended first class method. Note that within the extended first class method it is difficult to handle rigorously the anisotropic auto-correlation of soil properties, the general direction of the failure surface is usually considered in calculation of approximate correlation length corresponding to that direction.

3 LODALEN SLIDE

Slide in Lodalen marshalling yard near Oslo, Norway (Sevaldson 1956), has been chosen for the purpose of the evaluation of probabilistic numerical methods in this study. The slide is well documented, and it also served to El-Ramly et al. (2006) for evaluation of a simpler probabilistic approach to slope stability analysis, which may be compared with the present results.

The slide occurred in 1954 at the site where the marshalling yard was enlarged about 30 years ago. The inclination was approximately 1:2, as shown in Fig. 2, which depicts the mid-section through the slide with consecutive excavation steps. The main part of the slide is formed in comparatively homogeneous marine clay of sensitivity between 3 and 15 with some thin silt layers. In order to determine the causes of the slide, the Norwegian Geotechnical Institute performed a comprehensive series of borings for laboratory investigations and pore pressure measurements. Investigation of the samples allowed to determine probable location of failure surface, which had a rotational shape.

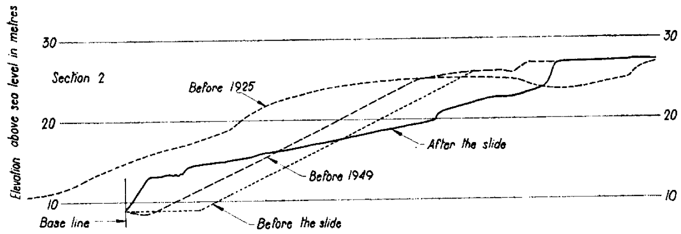

Figure 2. Mid-section through the Lodalen slide, from Sevaldson (1956)

The pore pressure measurements are shown in Fig. 3. The measurements show a definite increase of pore pressure with depth relative to the hydrostatic pressure distribution. There is thus an indication of artesian pressure in the ground, which can be explained by the rise of the country behind the slope.

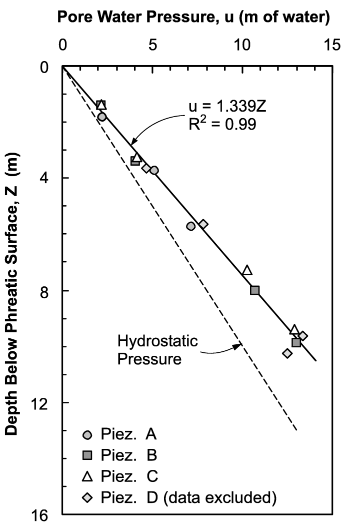

Figure 3. Pore pressures measured at the Lodalen site (El-Ramly et al., 2006; data from Sevaldson,1956)

The values of the effective friction angle φ and effective cohesion c have been found by means of undrained shear tests on different samples. Three or four undrained tests have been performed in order to construct each failure envelope. There was no marked difference between samples within and samples outside the slide. The statistical distributions of the measured parameters together with the gaussian fit are plotted in Fig. 4.

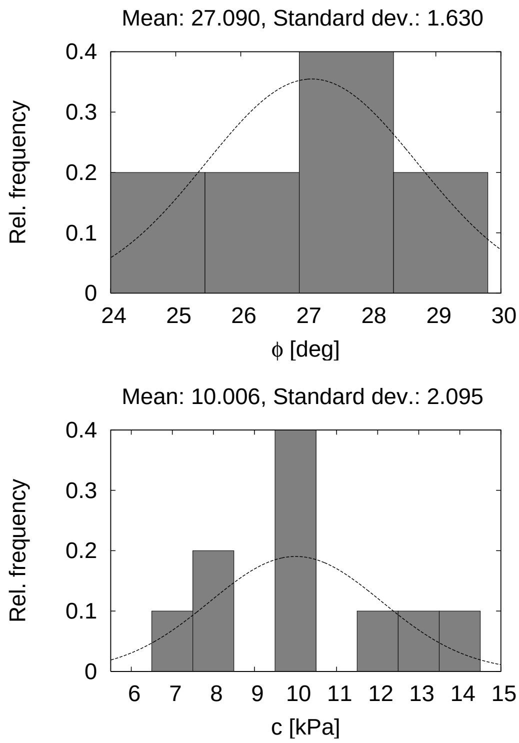

Figure 4. Statistical distributions of φ and c as measured by Sevaldson (1956)

4 FINITE ELEMENT SIMULATIONS

The problem has been simulated using deterministic finite element method, combined with the random field theory by Vanmarcke (1983). The finite element mesh, including dimensions, is shown in Fig. 5. The mesh represents the slope at the mid cross-section through the slide (Fig. 2), which is slightly steeper (5:9) than the overall slope of the slide (1:2). The mesh consists of 1123 9-noded isoparametric square elements, which reduce to triangles or irregular quadrilateral elements at the slant mesh boundary. Fig. 5 also shows assumed position of the water table, coming from piesometric measurements. The artesian pressure (Fig. 5) is modelled by increased unit weight of water. The slope is loaded by a gradual increase of the gravity acceleration until the failure occurs, which is indicated by a sudden increase of the slide mass velocity and impossibility to achieve convergence through the automatic time-stepping iterative procedure.

As indicated in Sec. 2, the random fields of two soil parameters that enter the calculation (φ and c) are generated by means of Cholesky decomposition technique. The random field is unconditioned (i.e. it does

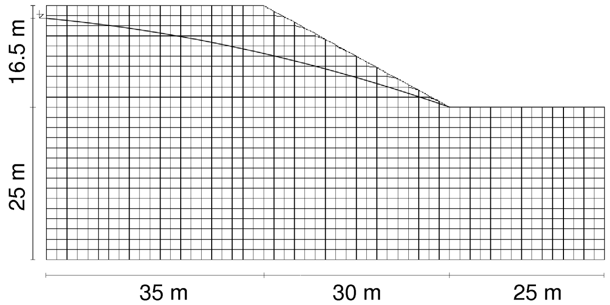

Figure 5. Finite element mesh used, with dimensions and assumed position of the ground water table.

not coincide with measurement results at exact measurement locations). The amount of data available is insufficient to produce a reliable conditioned random field. The statistical distribution of the output variable (gravity acceleration at failure) is found by means of the Monte Carlo method.

The two input random variables (φ and c) are described by gaussian statistical distributions, as shown in Fig. 4. As the experimental data show almost no cross-correlation between the two variables (cross-correlation coefficient between the two parameters is -0.0719), the two fields have been simulated as uncorrelated. In the present paper it is assumed that the distributions in Fig. 4 represent the inherent spatial variability of soil properties, i.e. error introduced through inaccurate laboratory procedures and uncertainty due to insufficient data available are neglected. For discussion of the latter two aspects, the reader is referred to Schweiger and Peschl (2005).

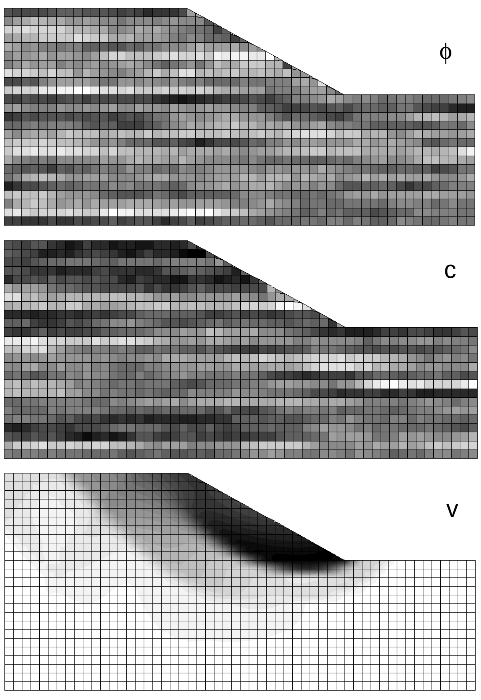

Figure 6. Typical realisation of random fields of uncorrelated variables φ and c for $\theta_h = 100$ m and $\theta_v = 1$ m, together with the velocity field at failure. Lighter areas of φ and c are softer.

As discussed in the Introduction, the last parameter of the random field, correlation length θ, is in general the most difficult to evaluate, and certainly it cannot be evaluated from the data available from the Lodalen slide site. For this reason, a parametric study on the influence of this uncertain parameter has been performed. In all cases, the same correlation length for both variables φ and c is considered. The horizontal correlation length θ_h took values 1, 10, 20, 50 and 100 m; and the vertical correlation length θ_v 1 and 10 m. Combination $\theta_h = 1$ m and $\theta_v = 10$ m has been disregarded as unrealistic. Typical realisation of the random fields of φ and c, for $\theta_h = 100$ m and $\theta_v = 1$ m, together with the velocity field at failure, is shown in Fig. 6.

In addition to the second class method calculations, two more simulations have been performed. One with the simpler first class method of Sec. 2 (infinite correlation length) and the second one with the extended first class method. In the latter case, an infinite correlation length is assumed for the random field realisation, whereas standard deviations of φ and c are reduced by the factor $\gamma = 0.206$ of Eq. (5) that corresponds to the integration of the Markov function (Eq. (3)) for $\theta = 10$ m along the potential failure surface of $L = 45$ m, which comes from the deterministic slope stability analysis.

5 PROBABILITY OF FAILURE

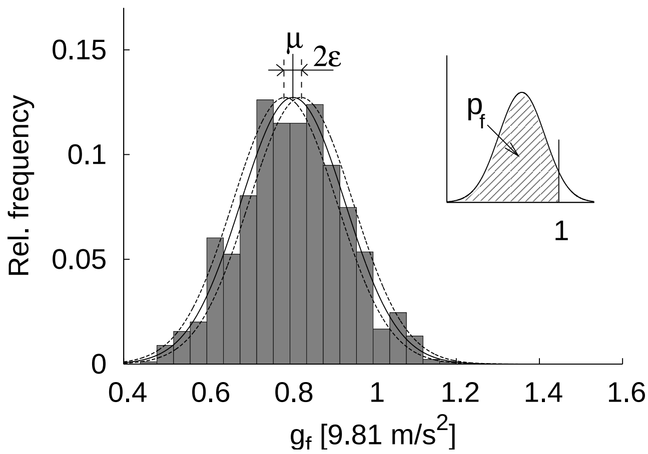

Figure 7. Evaluation of the probability of failure and 95.4 % confidence intervals of the Monte Carlo simulation from the statistical distribution of the gravity acceleration at failure g_f for the case $\theta_h = \theta_v = 10$ m.

The output variable coming from the Monte Carlo simulations is the gravity acceleration at failure g_f. Typical statistical distribution of this variable (for the case $\theta_h = \theta_v = 10$ m) is shown in Fig. 7. Clearly, the distribution of g_f fits closely the gaussian distribution, it is therefore possible to use the gaussian fit to calculate the probability of failure, which is equal to the area below the gaussian curve for $g_f < 1$ (as demonstrated in the nested image in Fig. 7). In this paper, the probability of failure has always been calculated from the gaussian fit, rather than from the ratio of the number of failed slopes to the total number of simulated slopes. Therefore, the "nominal" p_f is used instead of the "calculated" one, in the sense as defined by Wang and Chiasson (2006).

The fact that g_f follows the gaussian distribution also enables us to calculate the confidence intervals of the Monte Carlo outcome using the Chebychev inequality (Eq. (2)), as also demonstrated in Fig. 7. For the used number m of Monte Carlo realisations (typically, $500 < m < 1000$ in the present work) the uncertainty in determination of the mean value of g_f is for 95.4% confidence intervals rather small.

6 RESULTS OF SIMULATIONS

Figure 8 shows the nominal probability of failure p_f as a function of the correlation length, together with the 95.4% confidence intervals. For the second class models, there is a clear indication of increasing probability of failure with decreasing correlation length. The limit value with an infinite correlation length (first class model) gives approximately 20 % lower p_f. This demonstrates that disregarding the spatial correlation structure would lead to unconservative design. The graph also allows us to evaluate uncertainty in calculation of p_f as a function of uncertainty in the particular value of θ. Inaccurate assumption of θ, within the realistic bounds (Sec. 1), may lead to approximately 10% uncertainty in the calculated p_f. Any realistic guess of θ is therefore better than application of the first class model.

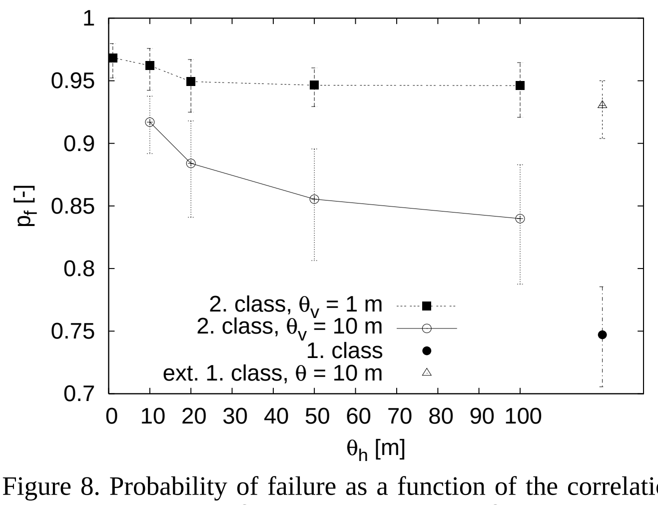

Figure 8. Probability of failure as a function of the correlation length. Results by the first class and extended first class methods included.

Figure 8 also shows results of the extended first class model for $\theta = 10$ m. Results are within Monte Carlo confidence intervals with the simulation by the second class model with $\theta_h = \theta_v = 10$ m. Incorporation of spatial averaging into the first class model thus leads in this case to significant improvement in predictions. This issue is discussed further in the following section.

7 DISCUSSION

From the nested image in Fig. 7 it is clear that the probability of failure is increased by both decrease of the mean value $\mu[g_f]$ and decrease of its standard deviation $\sigma[g_f]$. There are two conceptually different mechanisms, which induce differences in statistical distributions of g_f calculated by the first and second class models:

1. Incorporation of the spatially variable structure into the numerical simulation may lead to concentration of less competent materials into distinct zones. The failure mechanism than develops through these zones, which controll the overall behaviour of the structure. This mechanism would induce decrease of the $\mu[g_f]$ value.

2. In the case the first mechanism is not applicable (for example if the failure surface is pre-defined), the failure surface passes with the same probability softer and harder regions. The overall behaviour of the structure is then approximately controlled by the average values of the material parameters along the failure surface. These average values have lower variance when compared to the generic statistical distribution of the input variable, which is the idea employed in the extended first class models. The probability of failure is in this case increased by the decrease of $\sigma[g_f]$ under constant $\mu[g_f]$.

Figure 9 shows statistical distributions of g_f predicted by the first class, extended first class, and second class methods for $\theta = 10$ m. The three methods predict similar $\mu[g_f]$, the first class method predict significantly higher $\sigma[g_f]$ than the extended first class and second class methods. Investigation of Fig. 6 reveals that the failure surface has in the present case a regular shape, which is apparently not influenced by the spatial distribution of the soil properties. Therefore, the overall behaviour is in this case dominated by the second mechanism, which is also revealed in a good agreement between the extended first class and the second class models.

The inapplicability of the first mechanism may be explained by the use of the simple Mohr-Coulomb constitutive model, with c and φ as *uncorrelated* input random variables. As both the parameters influence the shear strength in a conceptually similar manner, the use of uncorrelated random fields of c and φ does not produce distinguished areas of lower strength, as the influence of variation of c and φ is statistically canceled. This unrealistic situation is caused by the

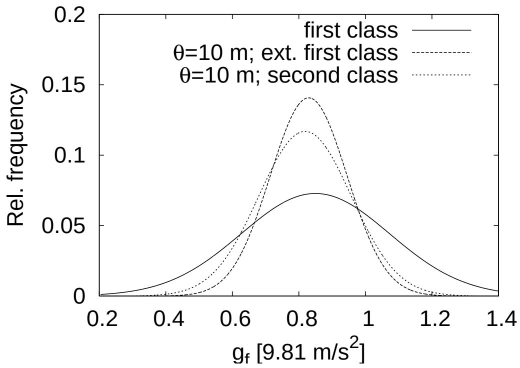

Figure 9. Distributions of g_f for the first class, extended first class, and second class methods for $\theta = 10$ m.

fact that the parameters φ and c, found by linearisation of a nonlinear failure envelope, do not represent sufficiently the soil behaviour. If enough data for model calibration are available, it is preferable to use advanced constitutive model, which relates the failure surface to other state variables (e.g., void ratio), as done within probabilistic framework by Niemunis et al. (2005) and Hicks and Onisiphorou (2005). For example, Hicks and Onisiphorou (2005) used an advanced constitutive model and showed that presence of pockets of highly liquifiable soil within the overall dense sand strata would lead to a significant reduction of the stability of the slope subject to dynamic loading, they thus demonstrated the importance of the first mechanism. The results presented in this paper show that care must be taken when simple constitutive models are used within advanced methods of numerical analysis.

8 CONCLUDING REMARKS

A particular slope in a fine-grained soil has been simulated using finite element method combined with the random field theory. It has been shown that for the soil described by the Mohr-Coulomb model with spatially variable uncorrelated parameters φ and c the correlation length θ influences the calculated probability of failure p_f. Not considering the random spatial structure would lead to unconservative design.

The influence of θ on p_f is pronounced although the given model does not allow to simulate concentration of less competent materials into distinct zones. Spatial averaging along the slip surface is the main factor dominating the slope behaviour and therefore simpler probabilistic methods with infinite θ and reduced variance may be used to simulate its behaviour. This result, however, would not be valid if more advanced material models were used.

9 ACKNOWLEDGEMENT

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REFERENCES

Christian, J. T. and G. B. Baecher (1999). Point-estimate method as numerical quadrature. *Journal of Geotechnical and Geoenvironmental Engineering 125*(9), 779–786.

Christian, J. T., C. C. Ladd, and G. B. Baecher (1994). Reliability applied to slope stability analysis. *Journal of Geotechnical Engineering ASCE 120*(12), 2180–2207.

Duncan, J. M. (2000). Factors of safety and reliability in geotechnical engineering. *Journal of Geotechnical and Geoenvironmental Engineering 126*(4), 307–316.

El-Ramly, H., N. R. Morgerstern, and D. M. Cruden (2003). Probabilistic stability analysis of a tailings dyke on presheared clay-shale. *Canadian Geotechnical Journal 40*, 192–208.

El-Ramly, H., N. R. Morgerstern, and D. M. Cruden (2006). Lodalen slide: a probabilistic assesment. *Canadian Geotechnical Journal 43*(9), 956–968.

Fenton, G. A. (1994). Error evaluation of three random field generators. *Journal of Engineering Mechanics ASCE 120*(12), 2487–2497.

Fenton, G. A. (1997). Probabilistic methods in geotechnical engineering. In *Workshop presented at ASCE GeoLogan'97 conference, Logan, Utah*.

Griffiths, D. V. and G. A. Fenton (2004). Probabilistic slope stability analysis by finite elements. *Journal of Geotechnical and Geoenvironmental Engineering 130*(5), 507–518.

Haldar, S. and G. L. S. Babu (2007). Effect of soil spatial variability on the response of laterally loaded pile in undrained clay. *Computers and Geotechnics (in print)*.

Hicks, M. A. and C. Onisiphorou (2005). Stochastic evaluation of static liquefaction in a predominantly dilative sand fill. *Géotechnique* (2), 123–133.

Niemunis, A., T. Wichtmann, Y. Petryna, and T. Triantafyllidis (2005). Stochastic modelling of settlements due to cyclic loading for soil-structure interaction. In G. Augusti, G. Schuëller, and M. Ciampoli (Eds.), *Proceedings of the 9th International Conference on Structural Safety and Reliability, ICOSSAR'05, Rome, Italy*. Millpress, Rotterdam.

Phoon, K.-K. and F. H. Kulhawy (1999). Characterisation of geotechnical variability. *Canadian Geotechnical Journal 36*, 612–624.

Popescu, R., J. H. Prevost, and G. Deodatis (1997). Effects of spatial variability on soil liquefaction: some design reccomendations. *Géotechnique 47*(5), 1019–1036.

Schweiger, H. F. and G. M. Peschl (2005). Reliability analysis in geotechnics with a random set finite element method. *Computers and Geotechnics 32*, 422–435.

Sevaldson, R. A. (1956). The slide in Lodalen, October 6th, 1954. *Géotechnique 6*, 167–182.

Singh, G. and C. M. Chung (1991). Microcompuetr-based reliability analysis in geotechnics. *Computers and Structures 41*(6), 1397–1402.

Vanmarcke, E. H. (1983). *Random fields: anaylisis and synthesis*. M.I.T. press, Cambridge, Mass.

Wang, Y.-J. and P. Chiasson (2006). Stochastic stability analysis of a test excavation involving spatially variable subsoil. *Canadian Geotechnical Journal 43*, 1074–1087.