MODELLING OF SHEAR MODULUS OF UNSATURATED FINE GRAINED SOILS AT VERY SMALL STRAIN

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ABSTRACT: *Shear modulus at very small strain is an important parameter in the design of geotechnical structures. The existing and new models have been evaluated using measured shear modulus at very small strain for a low plasticity fine grained soil available in the literature. It is found that the new model can be used to predict shear modulus at very small strain due to increase and decrease of mean net stress at constant suction. Moreover, the new model is able to predict a trend consistent with the experimental data for wetting-induced collapsible soil while existing models predict a contradictory trend.*

1 INTRODUCTION

Shear modulus at very small strain (0.001% or less), G_o is a key parameter in the design of geotechnical structures subjected to static and cyclic loadings. Although numerous researches have been conducted on the measurement of shear modulus at very small strain and empirical formulations were proposed and verified for saturated and dry soils, only recently investigation on G_o for unsaturated soil have been performed.

A few models have been proposed to predict shear modulus at very small strain for unsaturated soil. Some of them are simple semi-empirical formulations (Biglari et al., 2011; Leong et al., 2006; Ng and Yung, 2008; Sawangsuriya et al., 2009) while others involve complex formulation with larger number of variables (Biglari et al., 2011; Vassallo et al., 2007). Most of the existing models can predict shear modulus at very small strain due to increase of mean net stress at constant suction or increase of suction at constant mean net stress. Some of the existing models also include effects of stress history. However, most of the models cannot predict the shear modulus for wetting-induced collapsible soil. Recently, Wong et al. (2013) proposed a new model, which is simple but captures the effects of stress and suction as well as collapse during wetting on shear modulus at very small strain. A low plasticity fine grained soil data available in the literature are used to evaluate the existing and new models.

2 EXISTING MODELS FOR SHEAR MODULUS AT VERY SMALL STRAIN

Existing models used to predict shear modulus at very small strain can be divided into two primary groups depending on the stress variables used in the model. The first primary group adopts the following form.

$$G_o = \alpha f\left(p-u_a\right)g_1(s)+\beta g_2\left(s\right) \quad (1)$$

where mean net stress, $(p - u_a)$ and matrix suction, $s = (u_a - u_w)$ are used in the formulation. p, u_a and u_w are total mean stress, pore air and pore water pressures respectively. The first primary group can further be sub-divided into two categories depending on whether $\beta = 0$ or $\beta > 0$. Ng & Yung (NY) model proposed by Ng and Yung (2008) is under category with $\beta = 0$. NY model has the following equation for shear modulus at very small strain,

$$G_o = A\rho e^m\left[\left(\frac{p-u_a}{p_r}\right)^n\right]\left[1+\frac{s}{p_r}\right]^k \quad (2)$$

where bulk density, $\rho = (G_s+S_re)/(1+e)$. G_s, S_r and e are respectively specific gravity, degree of saturation and void ratio of the unsaturated soil. ρ_w is density of water. Sawangsuriya, Edil & Bosscher first (SEB1) model by Sawangsuriya et al. (2009) are under category with $\beta > 0$. SEB1 model has the following equation for shear modulus at very small strain,

$$G_o = Af_2(e)\left(p-u_a\right)^n + C\Theta^k s \quad (3)$$

in which

$$f_2\left(e\right)=\frac{1}{0.3+0.7e^2} \quad (4)$$

where C is a dimensionless parameter and $\Theta^k = S_r$.

The second of the primary groups adopts the following form:-

$$G_o = \alpha f\left[(p-u_a),s\right] \quad (5)$$

BMDJS model proposed by Biglari et al. (2011) has the following formulation which is based on the framework suggested by Gallipoli et al. (2003).

$$G_o = Ap_a^{1-n}f_1\left(e\right)OCR_{p'}^{m}\left(p'\right)^n h\left(S_r\right) \quad (6)$$

in which isotropic average soil skeleton stress is

$$p' = \left(p-u_a\right)+\chi s \quad (7)$$

where $\chi = S_r$. Function

$$h\left(S_r\right)=1-a'\left[1-\exp\left(b'\zeta\right)\right] \quad (8)$$

where a' and b' are dimensionless parameters and oversonsolidation ratio, $OCR_{p'} = \frac{p_o'\left(\zeta\right)}{p'}$.

Based on Gallipoli et al. (2003), $p_o'\left(\zeta\right)$ is obtained from

$$\ln p_o'(\zeta)=\frac{\lambda-\kappa}{\frac{e}{e_s}(\zeta)\lambda-\kappa}\ln p_o'(0)+\frac{\left(\frac{e}{e_s}(\zeta)-1\right)N}{\frac{e}{e_s}(\zeta)\lambda-\kappa} \tag{9}$$

in which

$$\frac{e}{e_s}(\zeta)=1-a\left[1-\exp(b\zeta)\right] \tag{10}$$

where $\zeta=1-S_r$ and $e_s=N-\lambda\ln p'$. e_s is the saturated void ratio measured during virgin compression with respect to the same average soil skeleton stress corresponding to the unsaturated void ratio, e. N is the intercept of the saturated normal compression line, λ is the slope of the saturated normal compression line and κ the saturated swelling index. $p_o'(0)$ is the isotropic average soil skeleton stress on the saturated normal compression line.

Wong et al. (2013) proposed a new model tackling the effects of stress and suction as well as collapse during wetting to shear modulus at very small strain. The formulation used in the new model is as follows,

$$G_o=\begin{Bmatrix} Ap_r^{1-n}e^m\left(p'\right)^n & s\le s_e \\ Ap_r^{1-n}e^m\left(p'\right)^n\left(\frac{s}{s_e}\right)^k & s>s_e \end{Bmatrix} \tag{11}$$

where the reference pressure, p_r is taken as 1 kPa. Following Khalili and Khabbaz (1998), the isotropic average soil skeleton stress can be represented by Equation 7, in which

$$\chi=\begin{Bmatrix} 1 & s\le s_e \\ \left(\frac{s_e}{s}\right)^\gamma & s>s_e \end{Bmatrix} \tag{12}$$

where γ is taken as 0.55 and s_e is the suction where the transition between saturated and unsaturated states occurs. For main drying path, $s_e = s_{en}$ while for main wetting path $s_e = s_{exp}$ in which s_{en} is air entry value and s_{exp} is air expulsion value. The effect of hydraulic hysteresis is considered in the formulation by normalising the suction with air entry value at main drying path or air expulsion value at main wetting path. At scanning curves in transition of drying to wetting or wetting to drying, the ratio at the main drying or wetting path is adopted.

3 MATERIAL AND MODEL CALIBRATION

In the following, we evaluate different models using experimental data on Zenoz kaolin by Biglari et al. (2012) and Biglari et al. (2011). Evaluations using other data sets can be found in Wong et al. (2013). Zenoz kaolin is an commercial Iranian kaolin. It is classified as CL according to the Unified Soil Classification System. Clay and silt fraction of Zenoz kaolin is about 18% and 60% respectively. Some of the properties for Zenoz kaolin are summarised in Table 1.

Table 1. Index properties of Zenoz kaolin used in the evaluation of models

Parameter	Value
Maximum dry density: kN/m^3	17.4
Optimum water content: %	15.4
Percentage of sand: %	22
Percentage of silt: %	60
Percentage of clay: %	18
Specific gravity	2.65
Liquid limit: %	29
Plastic limit: %	17
Plasticity index: %	12
Classification(USCS)	CL

There are four models used in the evaluation, namely NY, SEB1, BMDJS and the new models. For NY, SEB1 and BMDJS models, the variables are mean net stress, matrix suction, void ratio and degree of saturation. For the new model, suction at air expulsion or air entry is required instead of degree of saturation. The effect of void ratio on the air entry or air expulsion value of suction is not considered. According to Biglari et al. (2011), suction at air expulsion is 5 kPa. The suction at air entry is assumed to be two times of suction at air expulsion. For low plasticity clay, it is observed that suction at air entry is about two times of the suction at air expulsion from the soil water characteristic curve (SWCC) reported by Ng et al. (2009).

The calibration procedures for the models used in the evaluation are described in this section. The parameters A, m, n of NY and the new models are calibrated by fitting the shear modulus at very small strain during an isotropic loading-unloading test for a saturated soil. Parameter k is obtained using shear modulus during an isotropic loading test at a suction of 50 kPa. For SEB1 model, parameter n is taken as 0.5 as adopted in Sawangsuriya et al. (2009) and parameter A are obtained by fitting the shear modulus at very small strain during isotropic loading for saturated soils. Parameter C of SEB1 model is obtained by fitting the shear modulus at very small strain during isotropic loading at a suction of 50 kPa. Parameters for BMDJS model are obtained from Biglari et al. (2011). Model parameters for NY, SEB1, BMDJS and the new models are summarised in Table 2.

4 EVALUATION OF MODELS

Shear modulus at very small strain for commercially available Zenoz kaolin reported by Biglari et al. (2012) is used to evaluate the existing and proposed models. The specimen was prepared using moist tamping method at a water content of about 11.9%, which is 3.5% dry of the optimum from standard Proctor compaction test. The adopted method was intended to prepare samples, which could be brought to a virgin state at relatively low stress. The after compaction suction was 240 kPa. Subsequently, the samples were brought up to net mean stress of 50 kPa.

Fig. 1 shows the shear modulus at very small strain during isotropic loading-unloading test for a saturated soil. As BMDJS, NY and the new models are fitted to shear modulus during loading-unloading, the corresponding predictions agree well with the experimental data. Due to SEB1 model is calibrated using shear modulus during loading, thus a satisfactory prediction is expected. During unloading, SEB1 model underestimates the shear modulus.

The predicted shear modulus during unloading is slightly larger than those during loading attributed to an increase in the value of void ratio function as the soil become denser after loading.

Table 2. Model parameter values used by models for Zenoz kaolin

Parameter	NY model	SEB1 model	BMDJS model	New model
A	1150.21	3000	134.32	2176.12
M	-2.81	-	0.345	-3.05
N	0.380	0.5	0.626	0.375
K	0.260	-	-	0.370
C	-	750	-	-
N	-	-	0.997	-
λ	-	-	0.0725	-
κ	-	-	0.02	-
A	-	-	0.104	-
B	-	-	2.91	-
a'	-	-	0.143	-
b'	-	-	2.355	-
$s_{en:}$ kPa	-	-	-	10*
$s_{exp:}$ kPa	-	-	-	5

* assumed value

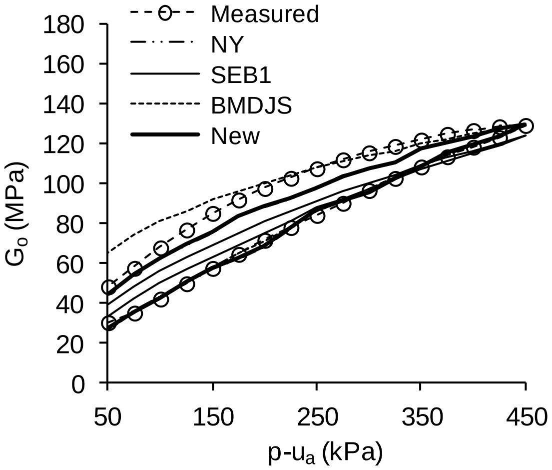

Fig. 1. Prediction of shear modulus at very small strain during isotropic loading-unloading cycle for saturated Zenoz kaolin

Fig. 2 compares the predicted and measured shear modulus at very small strain during an isotropic compression tests at matrix suction of 50 kPa. The specimen experiences plastic compression due to wetting-induced collapse when the after-compaction suction of 240 kPa is reduced to 50 kPa. As all the models are fitted to the measured shear modulus, the corresponding predictions agree well with the experimental data.

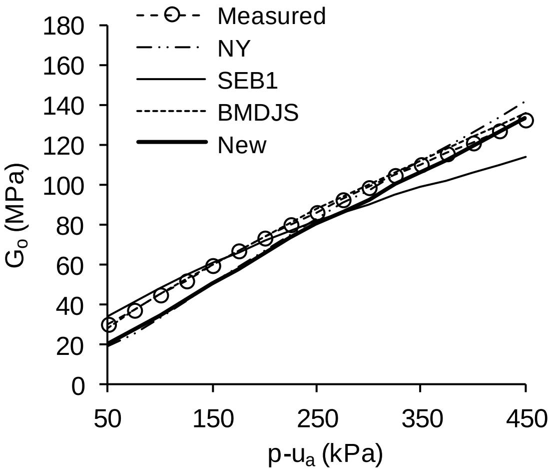

Fig. 2. Prediction of shear modulus at very small strain for unsaturated Zenoz kaolin during isotropic loading at suction of 50 kPa

Fig. 3 compares the predicted and measured shear modulus at very small strain during an isotropic loading-unloading test at matrix suction of 150 kPa. The specimen also experiences plastic compression due to wetting-induced collapse when the after-compaction suction of 240 kPa is reduced to 150 kPa. All the models predict an increasing trend with mean net stress consistent with experimental data. BMDJS and the new models give a good prediction on the shear modulus. NY model slightly underestimates but SEB1 model overestimates the shear modulus.

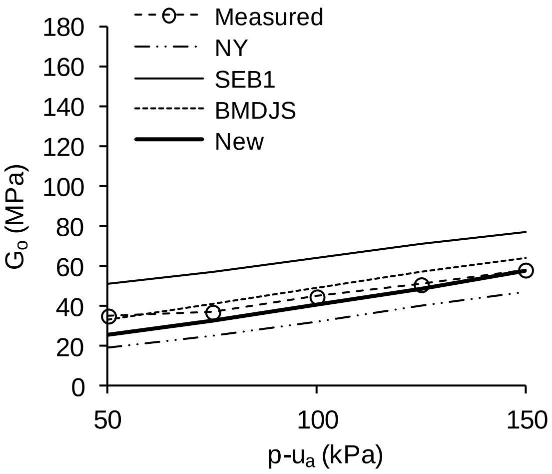

Fig. 3. Prediction of shear modulus at very small strain for unsaturated Zenoz kaolin during isotropic loading at suction of 150 kPa

Fig. 4 shows the variations of shear modulus at very small strain during loading-unloading cycle at suction of 300 kPa. All the models predict a trend and amount of hysteresis consistent with the experimental data. It is found that SEB1 model overestimates the shear modulus the most, while the other models are closer to the data. The difference in predictions is larger than those during isotropic loading test at suction of 150 kPa as suction of 300 kPa is further from the suctions used in the model calibrations.

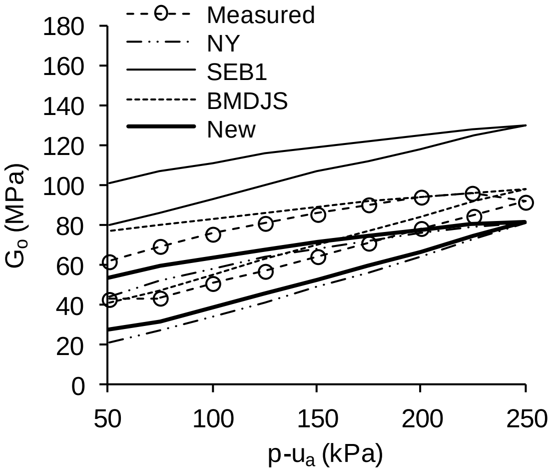

Fig. 4. Prediction of shear modulus at very small strain during isotropic loading-unloading cycle for unsaturated Zenoz kaolin at suction of 300 kPa

Fig. 5 compares the predicted and measured shear modulus at very small strain during wetting for a collapsible unsaturated soil. After equalization at mean net stress and matrix suction of 50 and 300 kPa respectively, the mean net stress is increased to 350 kPa. Under a constant mean net stress, the suction is reduced from 300 kPa to 50 kPa. NY, SEB1, BMDJS model predicts a decreasing trend when the suction was decreased from 300 to 50 kPa in contrast with the experimental data. For NY model, this suggests that the increase in shear modulus due to an increase in value of function ρe^m is unable to compensate the effect of suction reduction. For SEB1 model, this is because the increase in void ratio function does not compensate the reduction of shear modulus due to decreasing in function $S_r s$. For BMDJS models, this is because the increase in void ratio function does not compensate the reduction of shear modulus due to decreasing of average soil skeleton stress and function $h(S_r)$. The predicted shear modulus by the new model increases slightly as suction is reduced from 300 to 150 kPa. As the suction is further reduced to 50 kPa, there is a decreasing trend in the predicted shear modulus. The predicted trend is consistent with the experimental data. As the suction reduces from 300 kPa to 150 kPa, suction at air expulsion instead of air entry is adopted to normalise the suction and while the state lies at the scanning curve the ratio s/s_e is constant. The increase in predicted shear modulus attributed to the increase in void ratio function is more significant than the decrease due to a reduction of average skeleton stress. When the suction is further decreased to 50 kPa, there is a significant decrease in s/s_e value. This results in a decrease in the predicted shear modulus.

5 SUMMARY AND CONCLUSIONS

Several models from different categories, namely NY model proposed by Ng and Yung (2008), SEB1 model suggested by Sawangsuriya et al. (2009), BMDJS model presented by Biglari et al. (2011) and the new model proposed by Wong et al. (2013) have been evaluated. Shear modulus at very small strain for a low plasticity fine grained soils available in the literature are used in the evaluation. NY, BMDJS and new models can be used to predict shear modulus at very small strain in low plasticity fine grained soil due to increase and decrease of mean net stress at constant suction. It is found that NY, SEB1 and BMDJS models predict a contradictory trend with experimental data for wetting-induced collapsible unsaturated soil. Contrary, the new model is able to predict a trend consistent with the experimental data. The new model is evaluated in more detail in Wong et al. (2013).

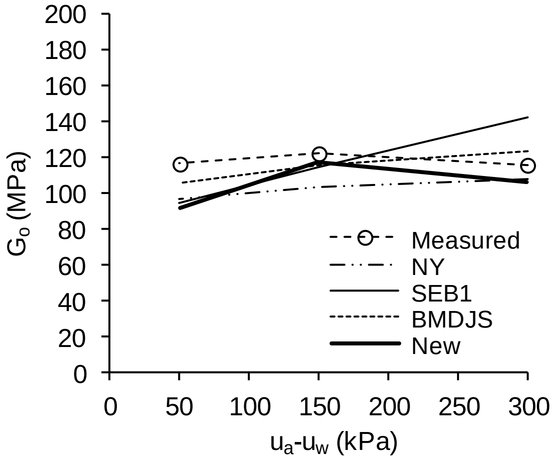

Fig. 5. Prediction of shear modulus at very small strain during wetting for normal consolidated unsaturated Zenoz kaolin

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