REVIEW ARTICLE: METHODS OF FRACTAL GEOMETRY USED IN THE STUDY OF COMPLEX GEOMORPHIC NETWORKS

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ABSTRACT

Fractal geometry methods allow one to quantitatively describe self-similar or self-affined landscape shapes and facilitate the complex/holistic study of natural objects in various scales. They also allow one to compare the values of analyses from different scales (Mandelbrot 1967; Burrough 1981). With respect to the hierarchical scale (Bendix 1994) and fractal self-similarity (Mandelbrot 1982; Stuwe 2007) of the fractal landscape shapes, suitable morphometric characteristics have to be used, and a suitable scale has to be selected, in order to evaluate them in a representative and objective manner.

This review article defines and compares: 1) the basic terms in fractal geometry, i.e. fractal dimension, self-similar, self-affined and random fractals, hierarchical scale, fractal self-similarity and the physical limits of a system; 2) selected methods of determining the fractal dimension of complex geomorphic networks. From the fractal landscape shapes forming complex networks, emphasis is placed on drainage patterns and valley networks.

If the drainage patterns or valley networks are self-similar fractals at various scales, it is possible to determine the fractal dimension by using the method "fractal dimension of drainage patterns and valley networks according to Turcotte (1997)". Conversely, if the river and valley networks are self-affined fractals, it is appropriate to determine fractal dimension by methods that use regular grids. When applying a regular grid method to determine the fractal dimension on valley schematic networks according to Howard (1967), it was found that the "fractal dimension of drainage patterns and valley networks according to Mandelbrot (1982)", the "box-counting dimension according to Turcotte (2007a)" and the "capacity dimension according to Tichý (2012)" methods show values in the open interval (1, 2). In contrast, the value of the "box-counting dimensions according to Rodríguez-Iturbe & Rinaldo (2001) / Kolmogorov dimensions according to Zelinka & Včelař & Čandík (2006)" was greater than 2. Therefore, to achieve values in the open interval (1, 2) more steps are needed to be taken than in the case of other fractal dimensions.

Keywords: fractal, drainage patterns, valley network, fractal dimension

1. Introduction

1.1 Introduction and objectives

Fractal aspects of complex nonlinear dynamic systems are ubiquitous in the landscape and in its studied phenomena (Table 1). Many natural features of the landscape have the appearance of a fractal; an example may be drainage patterns and valley networks or coast lines. Methods of fractal geometry have a mathematical basis which can be successfully applied in geomorphology. The behavior of complex natural phenomena, such as drainage systems, is at the forefront of research (Mandelbrot 1982; Voss 1988; Turcotte 1997, 2007a, 2007b; Bartolo & Gabriele & Gaudio 2000; Rodríguez-Iturbe & Rinaldo 2001; Saa et al. 2007; Stuwe 2007; Khanbabaei & Karam & Rostamizad 2013). Fractal dimensions and other fractal parameters in geomorphology are mainly used to quantitatively describe the topography of landscape fractal shapes and to build models of their development (Xu et al. 1993; Baas 2002).

In geomorphology, methods of fractal geometry were first applied in the study of the lengths of coastlines and the shape of drainage patterns and faults (Mandelbrot 1967; Robert 1988; Nikora 1991). Currently, fractal parameters have been used in geomorphology (Table 1): 1) while studying the spatial distribution of objects with different sizes (from microscopic to macroscopic objects); 2) while describing objects of intricate shapes (e.g. coral reefs, valley networks, mountains, caves, sand dunes); and 3) while studying processes and their areal distribution (e.g. erosion, chemical and mechanical weathering). Fractal geometry thus provides a way to quantitatively describe self-similar or self-affined landscape shapes, enables new approaches to measurements and analyses, and allows the holistic study of natural objects in various scales and a comparison of analysis values of different scales (Mandelbrot 1967; Burrough 1981).

When characterizing the fractal shape of complex geomorphic networks it is necessary to know and understand the basic concepts of fractal geometry, such as the fractal dimension, hierarchical scale, fractal self-similarity or physical boundary of the system. This work is based on a review of international and national literature in order to: 1) define and evaluate basic terms of fractal geometry which are applicable to the fractal shapes of complex geomorphic networks; and 2) define and evaluate certain methods of determining the fractal dimension of

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Tab. 1 Use of methods of fractal geometry in natural science (according to De Cola and Lam 2002a, 2002c).

Use of methods of fractal geometry in natural science			
Discipline	**Object of study**	**Discipline**	**Object of study**
Astronomy	Shape of Moon impacts; shape of galaxies	Botany	Shape of tree branches and roots
Geology	Thickness of layers of sedimentary rocks	Anatomy	Shape of vascular and nervous system, description of air sacks
Meteorology	Shapes of clouds, transfer of air temperature and water vapor	Ecology	Extension and concentration of pollution
Hydrology	Shape of drainage patterns, water surface	Landscape Ecology	Description of land cover
Geomorphology	Land surface, the extent of surface erosion	Cartography	Shape of coast and shoreline of lakes, map generalization

complex geomorphic networks. From the complex networks emphasis is placed in this research on drainage patterns and valley networks.

1.2 Definition of a fractal

The term fractal was first used by B. B. Mandelbrot (1967), who defined it as a set, whose fractal dimension is greater than its topological dimension (Table 2). The difference between the fractal and the topological dimension thus indicates the level of segmentation of a given object. The more the fractal dimension differs from the topological dimension, the more segmented an object is (Mandelbrot 1967). For example the shapes of drainage patterns or valley networks are made up of lines (topological dimension = 1), which are put on a plane (topological dimension = 2). The fractal dimension of the drainage patterns therefore describes to what extent the lines fill in the space on the plane and reach the values in the open interval (1; 2). The more the drainage pattern fills in the drainage basin, the more its fractal dimension approaches the value of 2 (Turcotte 1997).

Tab. 2 Definitions of terms of fractal geometry.

Author	**Definition**
Dimension	
Tichý (2012)	A dimension is a fundamental characteristic of geometrical shapes, which when scaling remains unchanged. A dimension can be generally expressed as: $N = k^D$ where k is the reduction ratio, N is the minimum number of reduced shapes that can cover the original shape, and D is the dimension. In other words: A) if a line is reduced k-times, then to cover the original segment $N = k^2$ new (reduced) lines are needed; B) if a rectangle is reduced k-times, then to cover the original rectangle $N = k^2$ new (reduced) rectangles are needed; C) if a cuboid is reduced k-times, then to cover the original cuboid $N = k^3$ new (reduced) cuboids are needed.
Initiator	
Horák & Krlín & Raidl (2007)	An initiator is the part of the shape, which is, under the construction of a fractal, replaced by a generator.
Generator	
Horák & Krlín & Raidl (2007)	A generator is the shape that under the construction of fractal, replace initiator, i.e. which forms the overall shape of the fractal object.
Topological dimension, also called the Lebesgue covering dimension	
Čech (1959); John (1978)	The topological dimension of n-dimensional Euclidean space is N. It is an integer dimension, which describes geometric objects. The topological dimension of a point = 0, the topological dimension of a line or curve = 1, the topological dimension of an area = 2. The topological dimension determines the minimum number of parameters needed to accurately determine the position of an object in the given space.
Fractal dimension, also called the Hausdorff–Besicovitch dimension	
Hausdorff (1919 in Mandelbrot 2003); Baas (2002); Tichý (2012)	A fractal dimension indicates the segmentation level of an object using a non-integer dimension. The shape of a valley network is formed by lines embedded in the plane, and the fractal dimension describes to what extent the space on the plane of the line is filled, thus reaching values in the open interval (1, 2).
Affine transformation	
Rodríguez-Iturbe & Rinaldo (2001); Turcotte (2007a)	Affine transformations include scale changing, i.e. resizing, rotation and displacement of the field, in which the fractal shape is captured.
Hausdorff measure	
Turcotte (2007a)	A Hausdorff measure is any number in the open interval (0, ∞) for each set of R^n, which has the role of a generator, i.e. forms an overall shape of a fractal object.

1.3 Definition of landscape shapes forming complex geomorphic networks

Landscape shapes, which are characterized by fractal geometry methods, include shapes forming complex geomorphic networks on the landscape, e.g. drainage patterns (Horton 1945), valley networks (Babar 2005), patterned ground polygons (Washburn 1979), or morphotectonic networks of lineaments (Kim et al. 2004). As watercourses join into drainage patterns, so the system of mutually interconnected valleys forms the valley networks, i.e. the system of linear depressions, each of which extends in the direction of its own thalweg (Davis 1913; Goudie 2004). The basic units of the drainage patterns are therefore watercourses, and the basic units of valley networks are thalwegs. The shapes and density of drainage patterns and valley networks are the result of the geomorphological development of the whole area and reflect the influence of the lithological-tectonic base (structure) and erosion on the formation of the landscape (Stoddart 1997).

Six basic shapes of valley networks have been distinguished (Howard 1967; Fairbridge 1968; Demek 1987; Babar 2005; Huggett 2007): 1) dendritic networks (they are often formed in areas with a low vertical division without the influence of structures); 2) parallel networks (they are often formed in areas with a considerable inclination of slopes or by the aggradation of large rivers; 3) trellis networks and 4) rectangular networks (they occur in areas with a dominant influence of continuous – folds and discontinuous – faults tectonic deformations); 5) radial networks (formed, for example, on volcanic cones); 6) annular networks (formed by destruction of vaults of sedimentary rocks).

1.4 Morphometric characteristics of complex geomorphic networks

Complex geomorphic networks can be presentable and objectively evaluated by morphometric characteristics. These characteristics describe hierarchical relations of units within the network and allow for a correlation between the sizes of several networks (Table 3) (Horton 1945; Babar 2005; Huggett 2007). For example, morphometric characteristics are commonly used in:

1) hydrology to describe drainage patterns (Horton 1945; Strahler 1957);

Tab. 3 Morphometric characteristics of valley networks according to Horton (1945), Turcotte (1997) and Mangold (2005).

Morphometric characteristics of valley networks		
Name	**Calculation**	**Definition**
Number of order X valleys n_X		It has been determined as the number of all order X valleys in the valley network.
Valley network density *D*	$D = L / P$	It has been determined as the ratio of the total lengths of thalwegs *L* to the valley network area *P*.
Frequency *F*	$F = N / P$	It has been determined as the ratio of the number of valleys *n* to the study area *P*.
Bifurcation ratio of valleys *Rb*	$Rb = n_X / n_{X+1}$	It indicates the rate of valley network branching. Where n_X is the "number of valleys of the given order" according to the Gravelius ordering system (Gravelius 1914) and n_{X+1} is the "number of valleys of one order higher" in the given valley network.
Total length of order X valleys t_X		It has been defined as the sum of lengths of all order X valleys in the valley network.
Total length-order ratio of valleys *T*	$T = t_{X+1} / t_X$	Where t_X is the "total lengths of valleys of the given order" according to the Gravelius ordering system (Gravelius 1914) and t_{X+1} is "the total length of valleys of one order higher" in the given valley network.
Average length of order X valleys l_X	$L_X = t_X / n_X$	Where t_X is the "total length of valleys of the given order" according to the Gravelius ordering system (Gravelius 1914) and n_X is the "number of valleys of the given order" in the given valley network.
average length-order ratio of valleys *Rr*	$Rr = l_X / l_{X+1}$	Where l_X is the "average lengths of valleys of the given order" according to the Gravelius order system (Gravelius 1914) and l_{X+1} is the "average valley length of one degree higher order" in the same network.
Fractal dimension of valleys *F*	$Fd = ln(Rb) / ln(Rr)$	Where *Rb* is the "bifurcation ratio of valleys" and *Rr* is the "average length-order ratio of valleys".
Valley junction angle		It express the angles at which the subsidiary (order X + 1) valleys run into the main (order X) valleys projected on a horizontal plane.
Frequency of valley junction angle *H*	$H = U / P$	It has been determined as the ratio of the number of valley junction angle *U* to the valley network area *P*.
Homogeneity of order X valleys		It has been defined by comparing the lengths of the longest and the shortest valleys of the given order. This characteristic is based on the analogy of homogeneity of the polygon lengths of the patterned ground. The valleys of a given order are homogeneous if the length of the longest order valley does not exceed three times the lengths of the shortest valley of the same order. If the valley network is not "homogeneous", it is designated as being "variable".

2) geomorphology to describe valley networks (Table 3; Turcotte 1997; Babar 2005), morphotectonic networks of lineaments (Ekneligod & Henkel 2006), or to describe patterned ground (Washburn 1979);
3) botany to describe leaf venation (Zalensky 1904);
4) transport geography to describe transport communications (Kansky 1963).

The most commonly used morphometric characteristics (Table 3) are based on the number of valleys, which are of course affected by hierarchical ordering – network order. In order to describe drainage patterns and valley networks, absolute and relative models of determining the network order system have been used. The absolute model, also called the Gravelius ordering system of drainage patterns (Gravelius 1914), describes the network away from the river mouth to the river springs (Figure 2A). The network is formed by the main/primary (order X) watercourse, into which the subsidiary/secondary (order X+1) watercourses flow, and into these watercourses later flow the tertiary (order X+2) watercourses, etc. (Gravelius 1914). After the watercourse division (order X), a watercourse of a higher order (X+1) begins from two watercourses above the river mouth, which has: A) a shorter length; B) a lower rate of flow; C) a greater angle towards the watercourse in front of the river point. By contrast, a watercourse of the same order (X) remains a watercourse which has: A) a greater length; B) a greater rate of flow; C) a smaller angle towards the watercourse in front of the river point (Gravelius 1914).

Relative models of network ordering systems describe the network away from the river springs to the estuary. 1st order watercourses are parts of the watercourse from the river springs to the first node, i.e. the confluence of watercourses in the network. The most commonly used relative network order systems are:

1) Horton ordering system of drainage patterns (Horton 1945), where by joining two watercourses of the same order X the watercourse below the node obtains the order X+1 (in the direction from the river springs to the estuary), and at the same time the watercourse above of the node (in the direction from the river springs to the estuary) changes from order X to order X+1 which has: A) a greater length; or B) a smaller angle against the watercourse in front of the node (Figure 2B);
2) Strahler ordering system of drainage patterns (Strahler 1957), where by joining two watercourses of the same order X the watercourse below the node (in the direction from the river springs to the estuary) obtains the order X+1, and where by joining two watercourses of different orders the watercourse below the node takes the number of the higher order of the watercourse above the node that is not increased (Figure 2C);
3) Shreve ordering system of drainage patterns (Shreve 1966), where an addition of orders occurs (Figure 2D) by the joining of two watercourses, i.e. the order of each watercourse within the network indicates the total number of river springs within the network above this watercourse (in the direction towards the river springs).

2. Methods

Technical publications dealing with general fractal geometry and the application of its methods in various fields of science were selected to define and evaluate the basic terms of fractal geometry. The terms of fractal geometry were defined for an example of drainage patterns and valley networks and subsequently the views by various authors on the river or valley networks were compared.

Various methods of determining the fractal dimension of networks were defined based on research of drainage patterns and valley networks. For each method the conditions of use were described and subsequently their advantages and disadvantages compared to the other mentioned methods were evaluated. To evaluate the fractal dimension calculations using regular grids the "fractal dimension of drainage patterns and valley networks according to Mandelbrot (1982)", the "box-counting dimensions according to Rodríguez-Iturbe & Rinaldo (2001) / Kolmogorov dimensions according to Zelinka & Včelař & Čandík (2006)", the "box-counting dimension according to Turcotte (2007a)" and the "capacity dimension according to Tichý (2012)" were applied to the schematic valley networks according to Howard (1967).

3. Results and discussion

3.1 Definitions of terms of fractal geometry

3.1.1 Self-similar and self-affined fractal

This is a large group of fractals, which is in particular used to describe and illustrate natural objects. The mathematical definition of self-similarity in the two-dimensional space is based on the relation of points F and F´, where F(x, y) is statistically similar to point F´(rx, ry), and where r is the affine transformation (Table 2; Turcotte 2007a). The self-similar fractals are isotropic, i.e. they have, in all respects, the same properties and the values of fractal parameters are logically not dependent on the orientation of x and y axes (Mandelbrot 1982, 2003; Rodríguez-Iturbe and Rinaldo 2001). Self-similar fractals are resistant to affine transformations, i.e. no matter how the cutout area, where the fractal landscape shape is displayed, will extend/diminish, rotate or shift, the fractal shape remains the same.

The mathematical definition of self-affinity in the two-dimensional space is based on the relationship of points F and F´, where F(x, y) is statistically similar to point F´(rx, r^{Ha}y), where r is an affine transformation and Ha is the Hausdorff measure (Table 2; Turcotte 2007a).

Self-affined fractals are not isotropic (Mandelbrot 1982, 2003), i.e. they do not have the same properties in all respects (Mandelbrot 1982, 2003) and the values of fractal parameters are dependent on the orientation of the x and y axes (Rodríguez-Iturbe and Rinaldo 2001). Self-affined fractals are not resistant against affine transformations, i.e. if the cutout of the area, in which the fractal landscape form is displayed, will increase/decrease, rotate or shift in any way, the fractal shape will change.

The authors' views on the shape of drainage patterns or valley networks differ in the world literature. Mandelbrot (1982) describes the drainage patterns as self-similar fractals by using Horton's laws (Horton 1945). Voss (1988) adapts the measurements and designates the drainage patterns as self-affined fractals. Kusák (2013) in his fractal analysis of the valley networks in the Ethiopian Highlands divides the shapes of valley networks into two groups: 1) the shapes defined by the relationship of the main valley and subsidiary valleys connected to it, i.e. dendritic, trellis and rectangular valley networks that meet the definition conditions of self-similarity; and 2) the shapes defined on the basis of mutual relation of several major valleys, i.e. parallel, radial and annular valley networks that meet the conditions of the self-affinity definition.

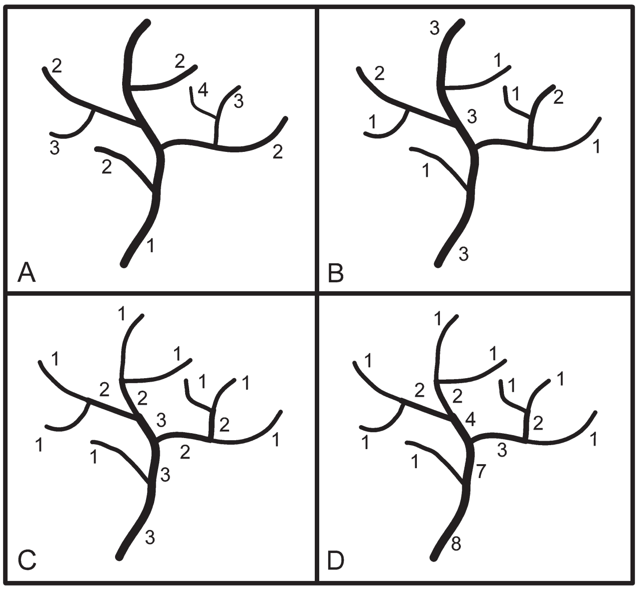

Fig. 1 Ordering systems of drainage patterns. Note: A – Gravelius ordering system of drainage patterns (Gravelius 1914); B – Horton ordering system of drainage patterns (Horton 1945); C – Strahler ordering system of drainage patterns (Strahler 1957); D – Shreve ordering system of drainage patterns (Shreve 1966).

3.1.2 Hierarchical scale, fractal self-similarity, physical limits of the system

At the beginning of each landscape research it is necessary to determine the scale to which the given shapes are described. When the map scale is increased (decrease in the size of pixel/picture element, decrease in the study area), a greater number of smaller shapes is shown on the map, e.g. cirques, etc. Such shapes are independent of each other and have a non-hierarchical scale (Bendix 1994).

Complex geomorphic networks consist of constantly recurring characteristic shapes, so called fractal self-similarity (Mandelbrot 1982; Stuwe 2007). The fractal shape can be divided into parts, each of which is (at least approximately) a copy of the whole shape. Fractal landscape shapes are thus defined in any resolution without giving the scale and their shape remains the same at any magnification or reduction (Baas 2002; Farina 2006). So the shapes in the given scale are affected by the whole of the superior scale and they alternatively influence the sub-whole of the hierarchically interior scale. According to Bendix (1994) the scale-independent shapes have a hierarchical scale. Self-similarity can in practice mean that when illustrating river drainage patterns without giving any scale, the flow of e.g. the Amazon is not recognizable from any other water course (Figure 1). Tarbotton

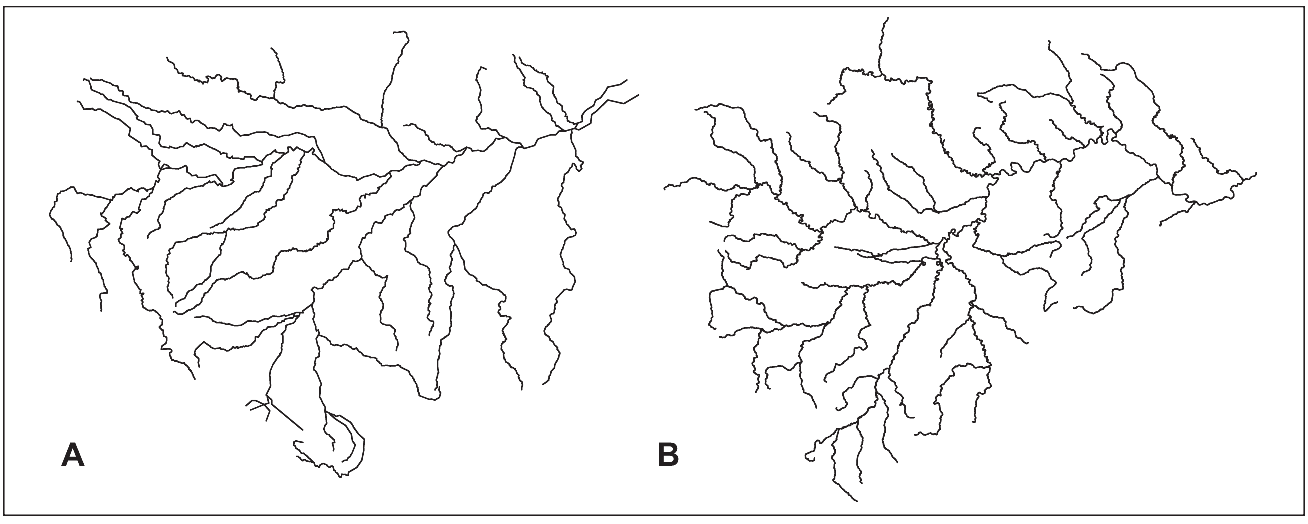

Fig. 2 Fractal self-similarity of drainage patterns. Note: A – Amazon drainage pattern (drainage basin 6,915,000 km^2); B – Berounka drainage pattern (drainage basin 8,855.47 km^2).

(1996) terms this property of fractal landscape shapes as the scale independence, Turcotte (1997, 2007a, 2007b) terms it as the scale invariance.

When measuring the length of a coast line it holds true that the length of the coastline increases with a more detailed scale (Mandelbrot 2003), i.e. the so-called Richardson effect (Zelinka & Včelař & Čandík 2006). In a mathematical sense, the geometrical structure in fractals is repeated up to infinity, i.e. the coastline would reach an infinite length at an infinitely large scale. With the fractal structure of landscape shapes there are certain boundaries that cannot be overcome, so called physical limits to the system. For example, according to Tichý (2012) the ratio between the largest and the smallest part of a fractal (self-similar) landscape shape is a maximum of 500 : 1. However, figure 2 shows that the ratio between the shape of the Amazon's drainage pattern and that of the Berounka is approximately 781:1. Due to the physical limits of the system, i.e. the limit that cannot be overcome in the landscape, geomorphology uses the fractal dimension of a final line (sensu Mandelbrot 2003).

3.2 Fractal dimension of drainage patterns and valley networks

3.2.1 "Fractal dimension of drainage patterns and valley networks according to Turcott (1997)"

Turcotte (1997, 2007a, 2007b) studied the use of fractals to describe the landscape and on the basis of bifurcation ratio Rb and the length-order ratio *Rr* (Table 2), he compiled a formula for calculating the fractal dimension *D* of drainage patterns and valley networks:

$D = ln(Rb) / ln(Rr)$.

The value of a fractal dimension indicates the extent to which the area is filled with watercourses or valleys. Increasing the value of the fractal dimension of drainage patterns of the order X + 1 means that the number of watercourses of the order X + 1 has increased or that the length of watercourses of the order X + 1 has increased, and the drainage patterns therefore fill the study area to a greater extent. The fractal dimension of drainage patterns and valley networks are different in various regions (due to the influence of the structural bedrock, tectonic activities) and even within a single region when changing the scale (Burrough 1981; Sung et al. 1998; Sung & Chen 2004).

However, Phillips (2002) describes the inaccuracies of the "fractal dimension of drainage patterns and valley networks according to Turcotte (1997)". The formula for calculating the fractal dimension is based on the bifurcation ratio and length-order ratio, which are based on the first and the second of Horton's law (Horton 1945). Horton's laws describe drainage patterns as self-similar fractals, i.e. he gives the same values of bifurcation ratios and length ratios between all orders. Real drainage patterns, however, are not self-similar (Voss 1988; De Cola & Lam 2002b). According to Phillips (2002), Horton's laws are more mathematical abstractions than the real state of the drainage patterns. Phillips (2002) conducted an analysis of the drainage patterns in the southern Appalachian Mountains with 30% of the drainage patterns having $Fd < 1$; 36% of the drainage patterns having $1 < Fd < 2$; and 34% of the drainage patterns having $Fd > 2$. The "fractal dimension of drainage patterns and valley networks according to Turcotte (1997)" is therefore not limited by an open interval (1; 2). Although the "fractal dimension of drainage patterns and valley networks according to Turcotte (1997)" is not limited by the open interval (1; 2), it is recognized in the world literature as a universal method for calculating the fractal parameters of drainage patterns and it is used most in geographic studies (e.g. Sung et al. 1998; Sung & Chen 2004; Turcotte 2007a, 2007b).

3.2.2 Determination of dimensions through the use of regular grids

Turcotte (2007a) studied self-affined fractal shapes and in determining the fractal dimension of shapes he overlaps these shapes with a regular grid, where each cell in the regular grid has dimensions *r* and *h*. Turcotte (2007a) gave an example of a self-affined fractal structure, where in the first step, the original shape of the line (indicator), which can be overlapped by just one cell, is divided into four lines (generator) that can be overlapped by four cells of a regular grid (Figure 3A, 3B). In the second and third

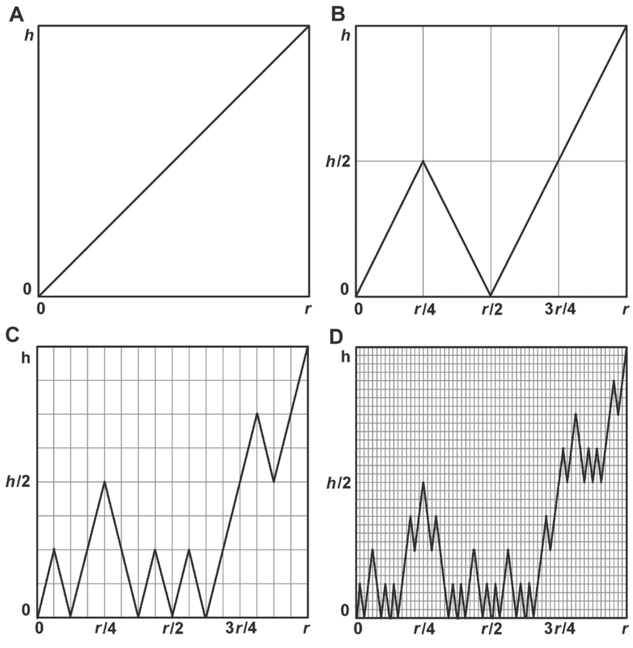

Fig. 3 Example of self-affined fractals according to Turcotte (2007a), modified. Note: A – zero initial condition of the shape of a self-affined fractal: an initiator, i.e. a straight line leading from point X(0, 0) to point Y(r, h), overlaid with one cell of a regular grid; B – the first step in the formation a self-affined fractal: generator, consisting of four lines, overlaid with four cells of regular grids; C – the second step in the formation of a self-affined fractal, overlaid with 16 cells of regular grids; D – the third step in the formation of a self-affined fractal, overlaid with 64 cells of regular grids.

step, each line (initiator) is likewise divided into four lines (generator), which can be overlapped exactly by sixteen (Figure 3C) and sixty four (Figure 3D) cells of a regular grid, respectively.

A regular grid can also be used in determining the size and shape complexity of complex geomorphic networks and their fractal dimension, e.g. Mandelbrot (1982), Rodríguez-Iturbe & Rinaldo (2001), Zelinka & Včelař & Čandík (2006), Turcotte (2007a) or Tichý (2012) (Table 4). A complex geomorphic network, such as a drainage pattern or valley network, is overlapped by a regular grid, the size of the cell side is usually defined in the interval r (0, 1) (Rodríguez-Iturbe & Rinaldo 2001). The cell size r in each step gradually decreases, thus the regular grid overlapping the drainage pattern or valley network becomes more detailed. The closer r is to 0, the more accurate the value of the box-counting dimension. The value of the fractal dimension is not dependent on the base of the logarithm (Table 4).

It was determined that the "fractal dimension of drainage patterns and valley networks according to Mandelbrot (1982)", the "box-counting dimension according to Turcotte (2007a)" and the "capacity dimension according to Tichý (2012)" reach values in the open interval (1, 2) (Table 4, Figure 5) in four steps using a regular grid, i.e. the first step $r_1 = 1$, the second step $r_2 = 0.5$, the third step $r_3 = 0.25$, and the fourth step $r_4 = 0.125$ (Table 4; Figure 4), on schematic valley networks according to

Tab. 4 The method of determining the fractal dimension by application of the regular grid by different authors and their application of schematic valley networks by Howard (1967).

Name	Calculation		Dendritic	Parallel	Trellis	Rectangular	Radial	Annular
"fractal dimension of drainage patterns and valley networks according to Mandelbrot (1985)"	$N_2 / N_1 = k^D$ After modification: $D = ln_{(k)} (N_2 / N_1)$ or $D = log_{(k)} (N_2 / N_1)$	D – fractal dimension; N_1 –number of cells covering drainage pattern and valley network with sizes x_1 and y_1; N_2 – number of cells covering drainage pattern and valley network with sizes $x_2 = kx_1$ and $y_2 = ky_1$, k – scaling factor, i.e. r_1/r_2, where r_1 – length of the cell side of the regular grid which covers drainage pattern and valley network with N_1 cells; r_2 – length of the cell side of the regular grid which covers drainage pattern and valley network with N_2.	$r_1 = 1$ $r_2 = 0.5$ $r_3 = 0.25$ $r_4 = 0.125$ $N_1 = 9$ $N_2 = 33$ $N_3 = 113$ $N_4 = 286$ $D_1 = 1.874$ $D_2 = 1.776$ $D_3 = 1.340$	$r_1 = 1$ $r_2 = 0.5$ $r_3 = 0.25$ $r_4 = 0.125$ $N_1 = 9$ $N_2 = 35$ $N_3 = 120$ $N_4 = 309$ $D_1 = 1.959$ $D_2 = 1.778$ $D_3 = 1.365$	$r_1 = 1$ $r_2 = 0.5$ $r_3 = 0.25$ $r_4 = 0.125$ $N_1 = 9$ $N_2 = 35$ $N_3 = 135$ $N_4 = 438$ $D_1 = 1.959$ $D_2 = 1.948$ $D_3 = 1.698$	$r_1 = 1$ $r_2 = 0.5$ $r_3 = 0.25$ $r_4 = 0.125$ $N_1 = 9$ $N_2 = 31$ $N_3 = 104$ $N_4 = 244$ $D_1 = 1.784$ $D_2 = 1.746$ $D_3 = 1.230$	$r_1 = 1$ $r_2 = 0.5$ $r_3 = 0.25$ $r_4 = 0.125$ $N_1 = 9$ $N_2 = 32$ $N_3 = 92$ $N_4 = 231$ $D_1 = 1.830$ $D_2 = 1.524$ $D_3 = 1.328$	$r_1 = 1$ $r_2 = 0.5$ $r_3 = 0.25$ $r_4 = 0.125$ $N_1 = 9$ $N_2 = 27$ $N_3 = 79$ $N_4 = 212$ $D_1 = 1.585$ $D_2 = 1.549$ $D_3 = 1.424$
"box-counting dimensions according to Rodríguez-Iturbe & Rinaldo (2001) / Kolmogorov dimensions according to Zelinka & Včelař & Čandík (2006)"	$D = ln\ N(r) / ln\ (1/r)$ or $D = log\ N(r) / log\ (1/r)$	D – box-counting dimension / Kolmogorov dimension; r – length of one cell side of the regular grid, which covers drainage pattern and valley network; $N(r)$ – number of cells of the regular grid, which covers drainage pattern and valley network. Calculation of ox-counting dimension is defined only for cell sizes lengths r (0; 1), and the closer the r is to 0, the value of box-counting dimension is more accurate.	$r_1 = 1$ $r_2 = 0.5$ $r_3 = 0.25$ $r_4 = 0.125$ $N_1 = 9$ $N_2 = 33$ $N_3 = 113$ $N_4 = 286$ D_1 – can not $D_2 = 5.044$ $D_3 = 3.410$ $D_4 = 2.720$	$r_1 = 1$ $r_2 = 0.5$ $r_3 = 0.25$ $r_4 = 0.125$ $N_1 = 9$ $N_2 = 35$ $N_3 = 120$ $N_4 = 309$ D_1 – can not $D_2 = 5.129$ $D_3 = 3.453$ $D_4 = 2.757$	$r_1 = 1$ $r_2 = 0.5$ $r_3 = 0.25$ $r_4 = 0.125$ $N_1 = 9$ $N_2 = 35$ $N_3 = 135$ $N_4 = 438$ D_1 – can not $D_2 = 5.129$ $D_3 = 3.538$ $D_4 = 2.925$	$r_1 = 1$ $r_2 = 0.5$ $r_3 = 0.25$ $r_4 = 0.125$ $N_1 = 9$ $N_2 = 31$ $N_3 = 104$ $N_4 = 244$ D_1 – can not $D_2 = 4.954$ $D_3 = 3.350$ $D_4 = 2.644$	$r_1 = 1$ $r_2 = 0.5$ $r_3 = 0.25$ $r_4 = 0.125$ $N_1 = 9$ $N_2 = 32$ $N_3 = 92$ $N_4 = 231$ D_1 – can not $D_2 = 5.000$ $D_3 = 3.262$ $D_4 = 2.617$	$r_1 = 1$ $r_2 = 0.5$ $r_3 = 0.25$ $r_4 = 0.125$ $N_1 = 9$ $N_2 = 27$ $N_3 = 79$ $N_4 = 212$ D_1 – can not $D_2 = 4.755$ $D_3 = 3.152$ $D_4 = 2.576$
"box-counting dimension according to Turcotte (2007a)"	$D = ln\ (N_2/N_1) / ln\ (r_1/r_2)$ or $D = log\ (N_2/N_1) / log\ (r_1/r_2)$	D – box-counting dimension; N_1 – number of cells covering drainage pattern and valley network with sizes r_1; N_2 – number of cells covering drainage pattern and valley network with sizes r_2.	$r_1 = 1$ $r_2 = 0.5$ $r_3 = 0.25$ $r_4 = 0.125$ $N_1 = 9$ $N_2 = 33$ $N_3 = 113$ $N_4 = 286$ $D_1 = 1.874$ $D_2 = 1.776$ $D_3 = 1.340$	$r_1 = 1$ $r_2 = 0.5$ $r_3 = 0.25$ $r_4 = 0.125$ $N_1 = 9$ $N_2 = 35$ $N_3 = 120$ $N_4 = 309$ $D_1 = 1.959$ $D_2 = 1.778$ $D_3 = 1.365$	$r_1 = 1$ $r_2 = 0.5$ $r_3 = 0.25$ $r_4 = 0.125$ $N_1 = 9$ $N_2 = 35$ $N_3 = 135$ $N_4 = 438$ $D_1 = 1.959$ $D_2 = 1.948$ $D_3 = 1.698$	$r_1 = 1$ $r_2 = 0.5$ $r_3 = 0.25$ $r_4 = 0.125$ $N_1 = 9$ $N_2 = 31$ $N_3 = 104$ $N_4 = 244$ $D_1 = 1.784$ $D_2 = 1.746$ $D_3 = 1.230$	$r_1 = 1$ $r_2 = 0.5$ $r_3 = 0.25$ $r_4 = 0.125$ $N_1 = 9$ $N_2 = 32$ $N_3 = 92$ $N_4 = 231$ $D_1 = 1.830$ $D_2 = 1.524$ $D_3 = 1.328$	$r_1 = 1$ $r_2 = 0.5$ $r_3 = 0.25$ $r_4 = 0.125$ $N_1 = 9$ $N_2 = 27$ $N_3 = 79$ $N_4 = 212$ $D_1 = 1.585$ $D_2 = 1.549$ $D_3 = 1.424$
"capacity dimension according to Tichý (2012)"	$D = ln\ (N) / ln\ (n)$ or $D = log\ (N) / log\ (n)$	D – capacity dimension; N – number of cells covering drainage pattern and valley netrosk; n – number of cells forming the site of regular grid.	$N_1 = 9$ $N_2 = 33$ $N_3 = 113$ $N_4 = 286$ $n_1 = 3$ $n_2 = 6$ $n_3 = 12$ $n_4 = 24$ $D_1 = 2.000$ $D_2 = 1.951$ $D_3 = 1.902$ $D_4 = 1.780$	$N_1 = 9$ $N_2 = 35$ $N_3 = 120$ $N_4 = 309$ $n_1 = 3$ $n_2 = 6$ $n_3 = 12$ $n_4 = 24$ $D_1 = 2.000$ $D_2 = 1.984$ $D_3 = 1.927$ $D_4 = 1.804$	$N_1 = 9$ $N_2 = 35$ $N_3 = 135$ $N_4 = 438$ $n_1 = 3$ $n_2 = 6$ $n_3 = 12$ $n_4 = 24$ $D_1 = 2.000$ $D_2 = 1.984$ $D_3 = 1.974$ $D_4 = 1.914$	$N_1 = 9$ $N_2 = 31$ $N_3 = 104$ $N_4 = 244$ $n_1 = 3$ $n_2 = 6$ $n_3 = 12$ $n_4 = 24$ $D_1 = 2.000$ $D_2 = 1.917$ $D_3 = 1.869$ $D_4 = 1.730$	$N_1 = 9$ $N_2 = 32$ $N_3 = 92$ $N_4 = 231$ $n_1 = 3$ $n_2 = 6$ $n_3 = 12$ $n_4 = 24$ $D_1 = 2.000$ $D_2 = 1.934$ $D_3 = 1.820$ $D_4 = 1.713$	$N_1 = 9$ $N_2 = 27$ $N_3 = 79$ $N_4 = 212$ $n_1 = 3$ $n_2 = 6$ $n_3 = 12$ $n_4 = 24$ $D_1 = 2.000$ $D_2 = 1.839$ $D_3 = 1.758$ $D_4 = 1.685$

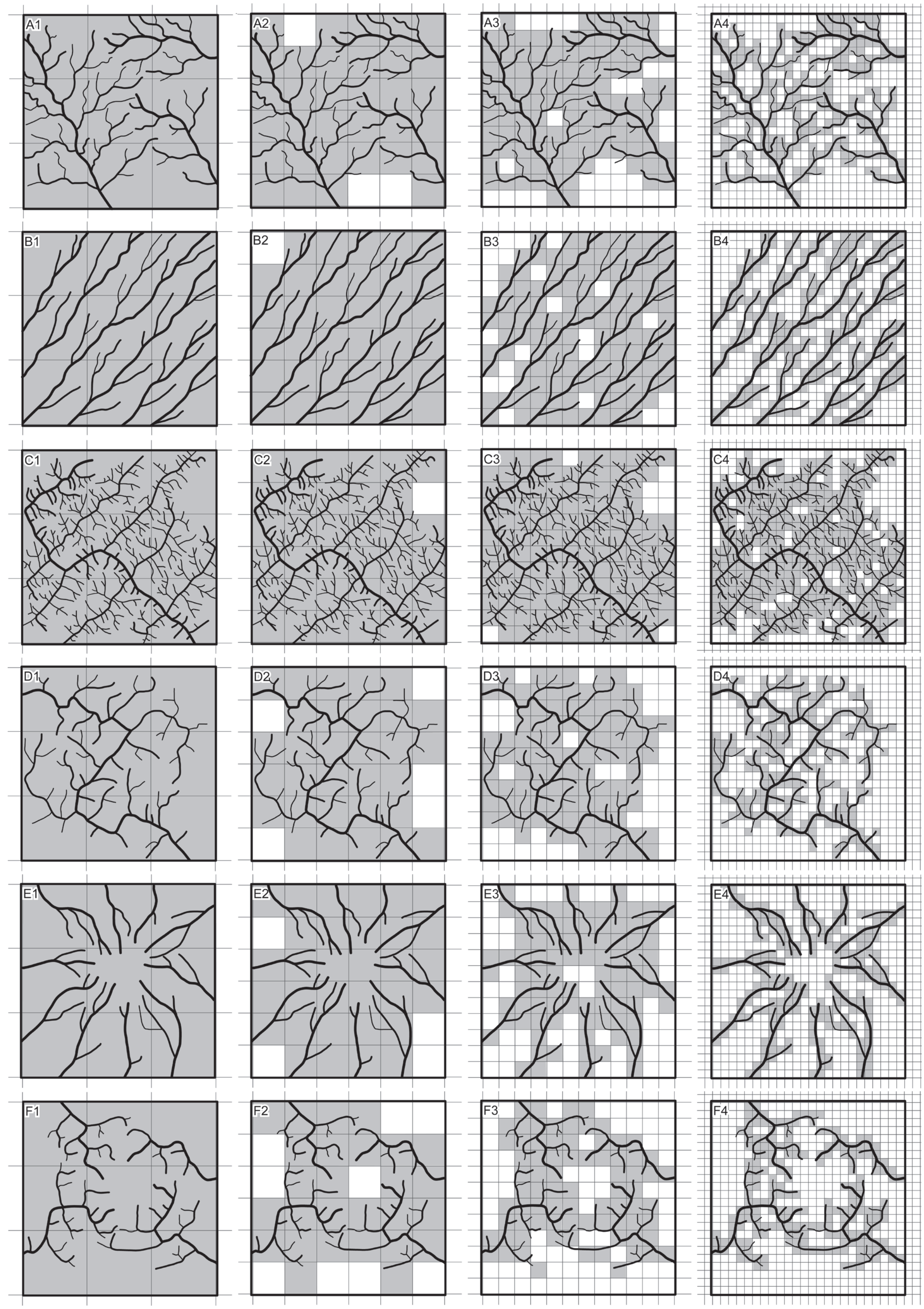

Fig. 4 Using a regular grid for the calculation of the fractal dimension of schematic valley networks according to Howard (1967). Note: A – dendritic valley network; B – parallel valley network; C – trellis valley network; D – rectangular valley network; E – radial valley network; F – annular valley network; 1 – the first step: $r_1 = 1$, N_1 (A, B, C, D, E, F) = 9; 2 – the second step: $r_2 = 0.5$, N_2 (A) = 33, N_2 (B, C) = 35, N_2 (D) = 31, N_2 (E) = 32, N_2 (F) = 27; 3 – the third step: $r_3 = 0.25$, N_3 (A) = 113, N_3 (B) = 120, N_3 (C) = 135, N_3 (D) = 104, N_3 (E) = 92, N_3 (F) = 79; 4 – the fourth step: $r_4 = 0.125$, N_4 (A) = 286, N_4 (B) = 309, N_4 (C) = 438, N_4 (D) = 244, N_4 (E) = 231, N_4 (F) = 212.

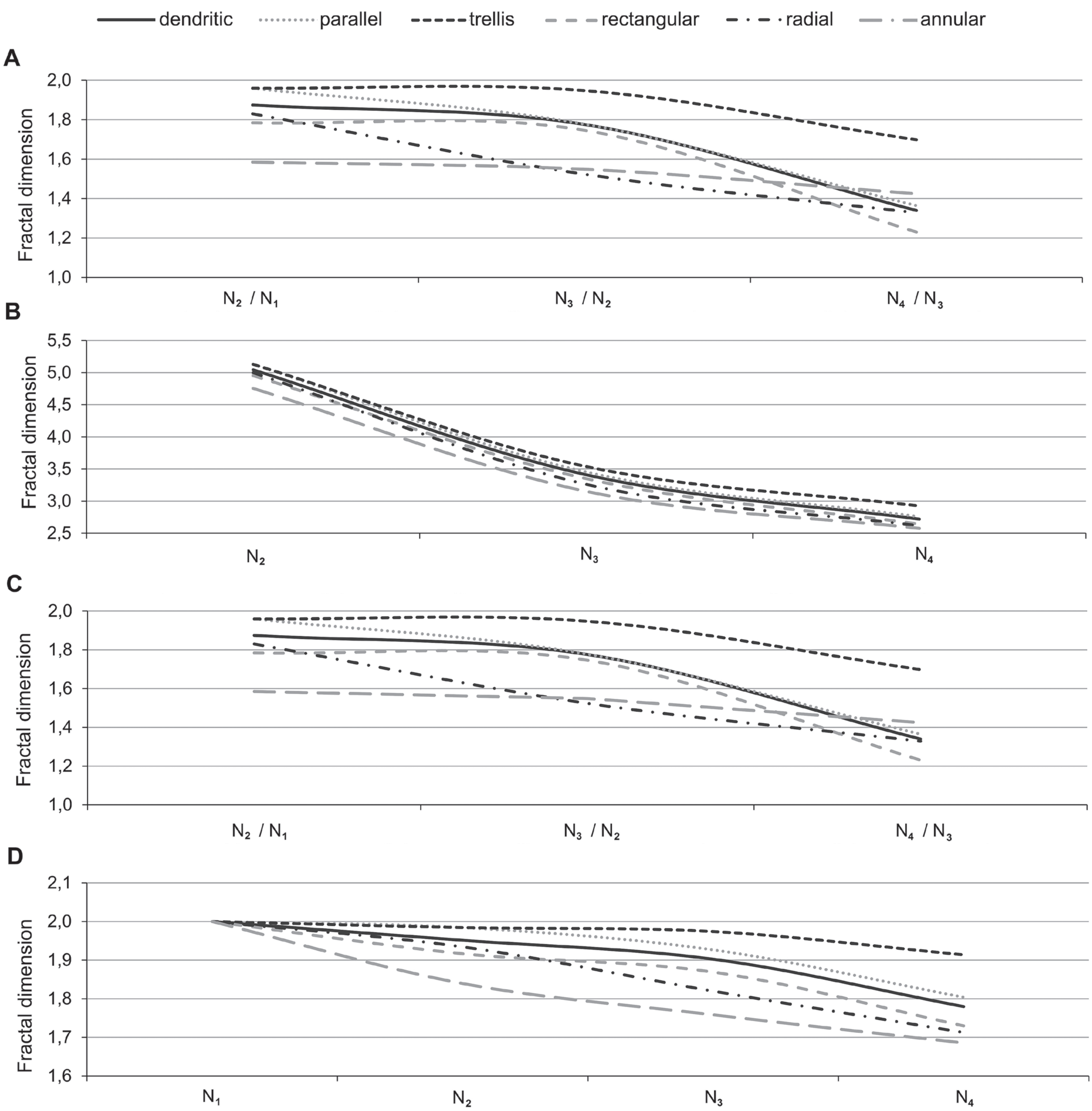

Fig. 5 Value of fractal dimensions applied to schematic valley networks according to Howard (1967). Note: A – "fractal dimension of drainage patterns and valley networks according to Mandelbrot (1982)"; B – "box-counting dimension according to Rodriguez-Iturbe & Rinaldo (2001) / Kolmogorov dimension according to Zelinka & Včelař & Čandík (2006)"; C – "box-counting dimension according to Turcotte (2007a); and D – "capacity dimension according to Tichý (2012)".

Howard (1967). This is in accordance with the definitions of a fractal dimension according to Hausdorff (1919 in Mandelbrot 2003), Baas (2002), and others. When Turcotte (2007a) defines the calculation of his "box-counting dimension", he refers to the definition of a "fractal dimension of drainage patterns and valley networks according to Mandelbrot (1982)", and although this calculation is adjusted in the four steps, the values of both dimensions are identical (Table 4; Figure 5). The values of the "box-counting dimension according to Rodriguez-Iturbe & Rinaldo (2001) / Kolmogorov dimension according to Zelinka & Včelař & Čandík (2006)", are greater than 2 (Table 4; Figure 5). In each further step the value of the dimension decreases. Thus, in order for the dimension value to reach values of an open interval (1, 2) more steps are required than for the other mentioned fractal dimensions.

3.2.3 Cellular automata

Fonstad (2006) studied the relations between landscape ecology and geomorphology and he studied fractal landscape shapes by means of so-called cellular automata. Cellular automata are used for modeling the time and space of fractal systems. The study area is divided into discrete cells (squares, triangles or hexagons), which form a regular grid (square, triangular or hexagonal), called a cellular network. The cell size is determined based on the parameters of a specific territory, i.e. it varies in different studies. Cells in the network have values according to whether or not they contain the studied fractal shape, i.e. if the value of the cell is 1 (black), the fractal shape is present but if the value of the cell is 0 (white), the fractal shape is not present. In each step, the cell values change depending on the value of the individual cells and their surroundings.

Cellular automata were first used in geomorphology by Barca et al. (1986) during the research of landslides and erosion. Afterwards cellular automata were used in other geomorphological studies, for example on the areal extent of erosion, the spatial distribution of aeolian sediments, or shapes of sand dunes. For the study of drainage patterns, cellular automata can be used only: 1) in semi-arid or arid areas where there are temporary streams (no surface runoff during the year); or 2) in areas where the bedrock is composed of unconsolidated rocks, that allow river braiding, and where the river easily and quickly relocates its riverbed. In such areas, the cell values in cellular automata may change and the changes of drainage patterns can be modeled using cellular automata. However, in most cases of drainage patterns and in all cases of valley networks, the use of cellular automata is not possible, since the cells in the grid should always have the same values. Despite the fact that in most cases of drainage patterns and in all cases of valley networks the use of cellular automata is not possible (because the cells in the grid have the same values), the "fractal cellar model according to Bi et al. (2012)" is considered to be inspirational and therefore will also be briefly analyzed.

3.2.4 "Fractal cellular model according to Bi et al. (2012)"

Bi et al. (2012) use a "fractal cellular model" to evaluate the fractal dimension of the landscape in the area of the Ordos Block (an area of 500,000 km^2 with located between the North China Platform and the Tibetan Plateau). This method can show the spatial variation of the fractal properties of the relief. It is a moving model, where "windows" of varying sizes are created which shift on the digital images of the area. The size of the squares sides W, which form a quadratic grid, is calculated from the relationship:

$$W = 2^n + 1,$$

where n is a positive whole number in the interval <1; 10>. If $n = 6$ m, then the size of the shifting "window" is 65×65 m. The "window" with a size of 65×65 m is shifted: 1) from the upper left corner of the study area to the bottom right corner; 2) only about 33 m, so that the segments of the area always partially overlap. Fractal parameters are then examined in the parts of the relief that capture the shifting "window". As with the cellular automata the areas in the "window" are designed as homogeneous units that can reach values of 1 (black) = there is a fractal shape and 0 (white) = there is not a fractal shape.

In general, calculating the size of the squares according to Bi et al. (2012) can also be applied for the study of other fractal landscape shapes. For example, when studying drainage patterns or valley networks, we can substitute n by the most numerous units in the network, i.e. the most frequent length of the rivers or valleys in the study area. In order to study the drainage patterns or valley networks, which consist of the largest number of 3 km long rivers or valleys, an area of 81 km^2 is ideal (sensu Bi et al. 2012). The fractal dimension can then be determined, for example using the "fractal dimension of drainage patterns and valley networks according to Turcotte (1997)", and then it is possible to compare how the value of the fractal dimension varies in different parts of the basin or when resizing the "windows".

4. Conclusion

Fractal landscape shapes are defined in any resolution without indicating the scale, i.e. the shape remains the same at any magnification or diminution (Baas 2002; Farina 2006), and they have a so-called hierarchical scale (Bendix 1994), where the shapes in the given scale are affected by the whole of the superior scale and they alternatively affect the subcomplex of a hierarchically lower scale. Self-similar and self-affined fractals are primarily used to describe and illustrate natural objects. Wherein, e.g. in determining the fractal shape of drainage patterns and valley networks, the results according to Mandelbrot (1982) and Turcott (1997), i.e. self-similar fractals, and according to Voss (1988), i.e. self-affined fractals, are different.

If the drainage patterns or valley networks are self-similar fractals, then the fractal dimension can be best determined using the "fractal dimension of drainage patterns and valley networks according to Turcotte (1997)". Although this is not limited by the open interval (1, 2) many authors use it as a universal method for calculating the fractal parameters and it is frequently used. If there is also an area of interest, i.e. a catchment area or area of the valley network which is divided into sub-areas, e.g. using the method according to Bi et al. (2012), the resulting value of the "fractal dimension of the drainage patterns and valley networks according to Turcotte (1997)" would be more accurate.

If the drainage patterns or valley networks are self-affined fractals, it is better to determine the fractal dimension by methods that use regular grids. When applying the method to determine the fractal dimension using a regular grid on a schematic valley network according to Howard (1967) it was determined that the "fractal dimension of drainage patterns and valley networks by Mandelbrot (1982)", "box-counting dimension according to

Turcotte (2007)" and "capacity dimension according to Tichý (2012)" show a value in the open interval (1, 2). In contrast, the value of "box-counting dimensions according to Rodríguez-Iturbe & Rinaldo (2001) / Kolmogorov dimensions according to Zelinka & Včelař & Čandík (2006)", was greater than 2, so to reach the values in the open interval (1, 2), more steps are needed than for the other fractal dimensions.

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RESUMÉ

Rešeršní článek: metody fraktální geometrie používané při studiích komplexních geomorfologických sítí

Metody fraktální geometrie umožňují kvantitativně popsat soběpodobné či soběpříbuzné tvary reliéfu, umožňují komplexní/holistické studium přírodních objektů v různých měřítkách a srovnání hodnot analýz z různých měřítek (Mandelbrot 1967; Burrough 1981). Vzhledem k hierarchickému měřítku (Bendix 1994) a fraktálové soběpodobnosti (Mandelbrot 1982; Stuwe 2007) fraktálních tvarů reliéfu tvořících složité sítě musejí být k jejich reprezentativnímu a objektivnímu zhodnocení použity vhodné morfometrické charakteristiky a zvoleno vhodné měřítko.

Tento rešeršní článek definuje a porovnává: 1) základní termíny fraktální geometrie, tj. fraktálová dimenze, soběpodobné, soběpříbuzné a náhodné fraktály, hierarchické měřítko, fraktální soběpodobnost a fyzikální hranice systému; a 2) vybrané metody určení fraktální dimenze geomorfologických komplexních sítí. Z fraktálních tvarů reliéfu tvořící komplexní sítě kladen důraz především na říční a údolní sítě.

Pokud říční či údolní sítě tvoří v různých měřítkách soběpodobné fraktály, je vhodné pro určení jejich fraktálních dimenzí užít „fraktální dimenze říčních a údolních sítí dle Turcotta (1997)". Naopak pokud říční či údolní sítě tvoří soběpříbuzné fraktály, je vhodné pro určení jejich fraktálních dimenzí užít metody využívající pravidelné mřížky. Při aplikaci metod určení fraktální dimenze pomocí využití pravidelné mřížky na schématické údolní sítě dle Howarda (1967) bylo zjištěno, že „fraktální dimenze říčních a údolních sítí dle Mandelbrota (1985)", „sčítací dimenze dle Turcotta (2007a)" a „kapacitní dimenze dle Tichého (2012)" dosahují hodnot v otevřeném intervalu (1; 2). Naopak hodnoty „sčítací dimenze dle Rodríguez-Iturbe & Rinalda (2001) / Kolmegorovovy dimenze dle Zelinky, Včelaře & Čandíka (2006)" byly větší než 2, čili pro dosažení hodnot v otevřeného intervalu (1; 2), je třeba více kroků než u ostatních fraktálních dimenzí.